Teacher Edition

Mathematics

Indiana's Academic Standards

www.indianastandards.org

Visit this Web site for hundreds of lesson plans and assessments aligned to the academic standards.

Adopted by the
Indiana State
Board of Education
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Introduction

The world we live in continues to change. For students to succeed in school, at work, and in the community, they will need more skills and knowledge than ever before. To ensure all students have every opportunity to succeed, Indiana adopted world-class academic standards in English/language arts, mathematics, science, and social studies and an assessment system to measure student progress toward the standards. These rigorous standards outline what students should know and be able to do at each grade level.

While the standards set expectations for student learning, they do not prescribe how the standards should be taught. Teachers should use their skills, experience, talents, and resources to design standards-based classroom lessons that meet the individual needs of their students.

Indiana's P-16 Plan for Improving Student Achievement

Indiana's academic standards are the cornerstone of the state's "P-16 Plan for Improving Student Achievement." Indiana's P-16 Plan provides a comprehensive blueprint for what educators, parents, and other adults must do to support students every step of the way – from their earliest years through post-high school education.

Indiana's World-Class Standards

Under the General Assembly's direction to develop standards that are "world-class, clear, concise, jargon-free, and by grade level," the standards were developed with the assistance of Indiana teachers, community members, and content experts at the university level.

Recommended by Indiana's Education Roundtable and adopted by the State Board of Education, Indiana's academic standards have been ranked among the best in the nation by Achieve, Inc., the Thomas B. Fordham Foundation, the International Center for Leadership in Education, the American Association for the Advancement of Science – Project 2061, and the National Council for History Education.

The Teacher Edition – Scope and Sequence

The Teacher Editions provide a complete set of Indiana's K-12 academic standards to ensure educators and administrators have full scope and sequence for curriculum alignment. Please note that definitions are provided throughout this document for explanatory purposes – it may not be appropriate to introduce technical definitions at lower grade levels.

The Importance of Parent and Student Involvement

Meeting higher expectations leads to greater rewards and opportunities for our students. We know that by setting specific goals, everyone wins. Teachers have clear targets, students know what's expected, and parents have detailed information about a child's strengths and weaknesses.

As a teacher, you know that parental involvement is vital to student success. The standards are a good way to engage parents in meaningful dialogue about student progress. It is also important to talk to students about these expectations – helping them take responsibility for their learning. More than simply a checklist, the standards provide a comprehensive look at what all students should know and be able to do at each grade level.

Encourage your students and their parents to review the academic standards online at www.learnmoreindiana.org.

Meeting the Challenge

The demand is greater than ever for people who can read, write, and speak effectively; analyze problems and set priorities; learn new things quickly; take initiative; and work in teams. Technology has already transported us into a time when opportunities are limited only by our imaginations. To keep our families, communities, and economy strong, all students need to keep learning after high school – at a two- or four-year college, in an apprenticeship program, or in the military.

With these academic standards in place, students in Indiana will be well-prepared to meet the challenges of the future.

For additional information and resources, such as classroom activities and assessments aligned to Indiana's academic standards at all grade levels, visit www.indianastandards.org.

Measuring Student Learning

Assessments help teachers and parents understand how students are progressing and identify academic areas where students may need additional attention. Assessments also provide a measure of school accountability – assisting schools in their efforts to align curriculum and instruction with the state's academic standards and reporting progress to parents and the public. Indiana's assessments are based on the state's academic standards and include the following:

- ■ **Indiana's Reading Assessments** are a series of informal classroom assessments available to assist teachers in Kindergarten through Grade 2. These optional assessments are designed to ensure students are learning to read at grade-level.
- ◗ **ISTEP+ Assessments** are given to students in Grades 3 through 10 in the fall of each school year. ISTEP+ measures what the child should have learned during the previous year. Results also are used to determine if schools are making adequate yearly progress in improving student achievement as part of Indiana's school accountability system under Public Law 221 and the federal No Child Left Behind Act of 2001.
- ◆ **Core 40 End-of-Course Assessments** are given in high school at the end of specific classes and are a cumulative test of what a student should have learned during that course. End-of-Course Assessments also provide a means to ensure the quality and rigor of high school courses across the state.

Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
■ Reading	■ Reading	■ Reading	◗ English/Lang. Arts ◗ Mathematics	◗ English/Lang. Arts ◗ Mathematics

What's the Goal? By Grade 4, have students moved beyond learning to read toward "reading to learn" other subjects? Can each student write a short, organized essay? Can each student use math skills to solve everyday, real-world problems?

Grade 5	Grade 6	Grade 7	Grade 8
◗ English/Lang. Arts ◗ Mathematics ◗ Science ◗ Social Studies (on hold – pending funding)	◗ English/Lang. Arts ◗ Mathematics	◗ English/Lang. Arts ◗ Mathematics ◗ Science ◗ Social Studies (on hold – pending funding)	◗ English/Lang. Arts ◗ Mathematics ◆ Core 40 End-of-Course Assessments

What's the Goal? By Grades 7 and 8, have students developed strong enough study habits in English and math skills to be ready for high school?

Grade 9	Grade 10 (GQE)	Grade 11	Grade 12	Graduation
◗ English/Lang. Arts ◗ Mathematics ◆ Core 40 End-of-Course Assessments	◗ English/Lang. Arts ◗ Mathematics ◆ Core 40 End-of-Course Assessments	(two retests available for those who have not passed the GQE) ◆ Core 40 End-of-Course Assessments	(two retests available for those who have not passed the GQE) ◆ Core 40 End-of-Course Assessments	(continued retesting opportunities available for those who have not passed the GQE)

What's the Goal? Can students read well enough to pass a driver's exam, understand an appliance manual, or compare two opposing newspaper editorials? Could students write an effective job application letter? By testing skills like these in Grade 10, teachers know whether – and in which skill area – students need more attention before it's time to graduate. By Grade 12, have students developed the academic foundation necessary to succeed in college and the workforce?

Visit **www.doe.state.in.us/standards/assessment.html** or call **1-888-54-ISTEP** for more information.

Indiana's Academic Standards for Mathematics

Preface for K-12 Teachers

Indiana's academic standards for mathematics represent a continuing effort to:

- ***Develop world-class standards for school mathematics;***
- ***Help all students achieve high standards by ensuring every school has a rigorous curriculum aligned with state standards and tests;***
- ***Benchmark Indiana's standards, assessments, and achievement against those of other states and nations.***

These standards were recommended by Indiana's Education Roundtable and adopted by the State Board of Education in Fall 2000 under the Indiana General Assembly's direction to develop standards that are "world class, clear, concise, jargon-free, and by grade level."

Noted as "among the clearest, most understandable, and most rigorous state standards in the nation" by Achieve, Inc., a nationally recognized independent organization that evaluates state academic standards, the new standards raise the expectations for student achievement previously set by Indiana's Mathematics Proficiency Guides of 1991 and 1997.

Teachers are expected to teach these standards. The statewide assessment of these standards was phased in for students in Grades 3, 6, and 8 in 2002 and for students in Grade 10 in 2004.

Indiana's academic standards for mathematics are based on the conceptual understandings embodied in the National Council of Teachers of Mathematics (NCTM) standards.

- Indiana's standards are presented by grade level through Grade 8 and by course through high school.
- Indiana's standards include seven content strands (number sense, computation, algebra and functions, geometry, measurement, data analysis and probability, and problem solving), with four process strands (communication, reasoning and proof, representation, and connections) embedded throughout.

Both the Indiana academic standards and the National Council of Teachers of Mathematics Standards may be effectively used together – the Indiana standards as a guide to set clear expectations for students and the NCTM Standards as a guide for teachers to develop students' conceptual understanding.

The Indiana Council of Teachers of Mathematics (ICTM) and the Indiana Mathematics Initiative (IMI) recommend use of NCTM Principles and Standards to Indiana teachers as a basis for effective implementation of Indiana's math standards. A synopsis of these Principles and Standards written by ICTM and IMI is below. The complete NCTM document is accessible through the World Wide Web at www.nctm.org.

National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics

The six **NCTM Principles** describe particular and important features of high-quality mathematics education. The ten **NCTM Standards** describe the mathematical *content* and *processes* that students should learn. Together, the Principles and Standards constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems.

Principles (NCTM)

The Equity Principle

Excellence in mathematics education requires equity – high expectations and strong support for all students.

All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics. This does not mean that all students should be treated the same. But all students need access each year they are in school to a coherent, challenging mathematics curriculum that is taught by competent and well-supported teachers. Equity does not imply lowering expectations for any group of students. Rather expectations must be raised for all – mathematics can and must be learned by all students. There is no conflict between equity and excellence.

The Curriculum Principle

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

School mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students. Mathematics topics can be considered important for different reasons, such as their utility in developing other mathematical ideas, in linking different areas of mathematics, or in deepening students' appreciation of mathematics as a discipline. A coherent curriculum allows students to effectively organize and integrate important mathematical ideas.

The Teaching Principle

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn.

Teachers have different styles and strategies for helping students learn particular mathematical ideas, and there is no one "right way" to teach. However, regardless of style, teachers must decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them. Selecting and using suitable curricular material, using appropriate instructional tools and techniques, and engaging in reflective practice and continuous self-improvement are actions good teachers take every day.

A good curriculum is not sufficient for effective learning and teaching. Teachers are required each day to make choices about how the learning environment will be structured and what mathematics will be emphasized. These decisions determine, to a large extent, what students learn. Effective teaching conveys the belief that each student can understand mathematics and that each will be supported in accomplishing this goal.

The Learning Principle

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Learning with understanding is essential to enable students to solve the new kinds of problems they will inevitably face in the future. Unfortunately, learning without understanding has long been a common outcome of school mathematics. Students who memorize facts and procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile.

Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways. Learning with understanding also makes subsequent learning easier. Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills.

The Assessment Principle

A*ssessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

Assessment should be more than merely a test at the end of instruction to gauge learning. It should be an integral part of instruction that guides teachers and enhances students' learning.

Teachers should be continually gathering information about their students through questions, interviews, writing tasks, and other means. They can then make appropriate decisions about such matters as reviewing materials, re-teaching a difficult concept, or providing something more or different for students who are struggling or need enrichment.

To be consistent with the Learning Principle, assessments should focus on understanding as well as procedural skills. Because different students show what they know and can do in different ways, assessments should also be done in multiple ways, such as open-ended questions, constructed-response task, selected-response items, performance tasks, observations, conversations, journals, and portfolios. Constructed-response or performance-tasks may better measure a student's capacity to apply mathematics in complex or new situations.

The Technology Principle

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Calculators and computers are reshaping the mathematics landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction. Every student should have access to appropriate technology to facilitate his or her mathematics learning.

Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies.

Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking.

Standards for School Mathematics

The **Standards** describe a connected body of mathematical understandings and competencies, and are a comprehensive foundation recommended for all students. They describe the mathematical understanding, knowledge, and skills that students should acquire from kindergarten through grade 12.

The Content Standards – *Number and Operations, Algebra, Geometry, Measurement, Data Analysis and Probability* – reflect the important mathematical content that should serve as the basis of Indiana School mathematics, and explicitly describe the content students should learn. The five Process Standards – *Problem Solving, Communication, Reasoning and Proof, Connections, and Representation* – highlight important and crucial ways of acquiring and using content knowledge. The Process Standards are not isolated components of the curriculum, but should be used in the study of the Content Standards. Students will attain, and be assessed on, the process standards while pursuing the various content standards. Problem solving is a central process standard and students will learn many process standards within the context of the problem-solving standard.

Content Standards

Number and Operations

Number pervades all areas of mathematics. The other four Content Standards as well as all five Process Standards are grounded in understanding number. Central to this standard is the development of number sense, which allows students to naturally combine or decompose numbers, solve problems using the relationships among operations and knowledge of the base-ten system, and make a reasonable estimate for the answer to a problem.

Computational fluency – having and using efficient and accurate methods for computing – is essential. Students should be able to perform computations in different ways, including mental calculations, estimation, and paper-and-pencil calculations using mathematically sound algorithms. All students should use calculators at appropriate times, setting the calculator aside when the instructional focus is on developing computational algorithms.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- compute fluently and make reasonable estimates.

Algebra

The ideas of algebra are a major component of the school mathematics curriculum and help to unify it. Mathematical investigations and discussions of arithmetic and its properties frequently include aspects of algebraic reasoning. Such experiences present rich contexts and opportunities for enhancing mathematical understanding and are an important precursor to the more formalized study of algebra in the middle and secondary grades. A strong foundation in algebra should be in place by the end of the eighth grade, and all high school students should pursue ambitious goals in algebra.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Geometry

Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science. Geometry is a natural area of mathematics for the development of students' reasoning and justification skills that build across the grades.

Geometry should be learned using concrete models, drawings, and dynamic software. As the study of the relationships among shapes and their properties becomes more abstract, students should come to understand the role of definitions and theorems and be able to construct their own proofs.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

The study of measurement is crucial in the K-12 mathematics curriculum because of its practicality and pervasiveness in many aspects of everyday life. Measurement is possibly the area of mathematics that is most important when considering everyday applications of mathematics, and highlights connections between mathematics and areas outside of the school curriculum such as social studies, science, art, and physical education. The study of measurement helps students establish connections within mathematics and provides an opportunity for learning about and unifying ideas concerning number and operations, algebra, geometry, statistics, probability, and data analysis.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability

To analyze data and reason statistically are essential to be an informed citizen, employee, and consumer. The amount of statistical information available to help make decisions in business, politics, research, and everyday life is staggering. Through experiences with the collection and analysis of data, students can learn to make sense of and interpret information and allow them to make appropriate arguments and recognize inappropriate arguments as well.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

Process Standards

Problem Solving

Problem solving means engaging in a task for which the solution process is not known in advance. Good problem solvers have developed a "mathematical disposition" which allows them to analyze situations in mathematical terms. They have developed a range of strategies for developing a solution to a problem, have learned to monitor and adjust the strategies they choose to use in the process of solving a specific problem, and can compare and contrast solutions and problems.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Communication

As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking clearly and coherently to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Reasoning and Proof

Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. By the end of secondary school, students should be able to understand and produce some mathematical proofs – logically rigorous deductions of conclusions from mathematical hypotheses – and should appreciate the value of such arguments.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and proof.

Connections

Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

NOTES

Kindergarten

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Kindergarten:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students develop this understanding by first comparing the number of objects (such as blocks) in a given set. From comparing sets of objects, they develop the concept of counting: matching each object in a set with a counting number. Then they use counting to recognize, name, and order up to ten objects. As preparation for learning about fractions, students practice dividing sets into equal groups and shapes into equal parts.

Standard 2 — Computation

Fluency in computation is essential. As students learn about numbers, they also learn how to add and subtract them. They use objects to join sets together (for addition) and to remove objects from sets (for subtraction).

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level sort and classify objects according to various rules and make simple patterns with numbers and shapes.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They identify and describe simple shapes, comparing and sorting them by such attributes as size and roundness. They learn the meaning of words, like inside and above, that relate to positions in space.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students begin their study of measurement by comparing objects' length, weight, temperature, etc. They use words like shorter, taller, heavier, and colder. They also learn concepts of time, such as hours, days, months, and years.

Standard 6 — Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with numbers, geometry, or measurement, for example, students at this level move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, $\frac{1}{4}$, one divided by four, 0.25, $\frac{1}{8} + \frac{1}{8}$, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: $+$, $=$, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense

Students understand the relationship between numbers and quantities up to 10, and that a set of objects has the same number in all situations regardless of the position or arrangement of the objects.*

K.1.1 Match sets of objects one-to-one.

Example: Take crayons from the box and give one to each student in the group. Explain what you are doing.

K.1.2 Compare sets of up to ten objects and identify whether one set is equal to, more than, or less than another.

Example: Compare the blocks in two boxes. Tell which box contains more blocks and explain the way in which you decided on your answer.

K.1.3 Know that larger numbers describe sets with more objects in them than sets described by smaller numbers.

Example: Understand that a set of 7 apples contains more apples than a set of 3 apples.

K.1.4 Divide sets of ten or fewer objects into equal groups.

Example: Take 6 blocks and give the same number to each of 3 children.

K.1.5 Divide shapes into equal parts.

Example: Divide a piece of paper into 4 equal pieces.

K.1.6 Count, recognize, represent, name, and order a number of objects (up to 10).

Example: Count a group of seven pennies. Recognize that 7 is the number for this set.

K.1.7 Find the number that is one more than or one less than any whole number* up to 10.

Example: You have a bag of 7 apples. How many apples are in a box that holds one less than the bag of apples?

K.1.8 Use correctly the words *one/many*, *none/some/all*, *more/less*, and *most/least*.

Example: Take some of the blocks out of this box, but not all of them.

K.1.9 Record and organize information using objects and pictures.

Example: Ask some of your friends what pets they have. Use pictures of animals to show the number of pets your friends have.

* set: a collection of objects, numbers, etc.

* whole number: 0, 1, 2, 3, etc.

Standard 2

Computation

Students understand and describe simple additions and subtractions.

K.2.1 Model addition by joining sets of objects (for any two sets with fewer than 10 objects when joined).

Example: Put together 3 pencils and 2 pencils. Count the total number of pencils.

K.2.2 Model subtraction by removing objects from sets (for numbers less than 10).

Example: From a pile of 9 crayons, take away 6 crayons. Count the number of crayons left in the pile.

K.2.3 Describe addition and subtraction situations (for numbers less than 10).

Example: In the last example, explain what operation you were using when you took away crayons from the pile.

Standard 3

Algebra and Functions

Students sort and classify objects.

K.3.1 Identify, sort, and classify objects by size, number, and other attributes. Identify objects that do not belong to a particular group.

Example: Find the squares in a collection of shapes. Sort these squares into large ones and small ones and explain how you decided which squares went in each pile.

K.3.2 Identify, copy, and make simple patterns with numbers and shapes.

Example: Make a pattern of squares and circles with one square, one circle, one square, one circle, etc.

Standard 4

Geometry

Students identify common objects around them and describe their geometric features and position.

K.4.1 Identify and describe common geometric objects: circle, triangle, square, rectangle, and cube.

Example: Look for cubes and circles at home and at school.

K.4.2 Compare and sort common objects by position, shape, size, roundness, and number of vertices.

Example: Compare the numbers of vertices of triangles, squares, and rectangles.

K.4.3 Identify and use the terms: *inside*, *outside*, *between*, *above*, and *below*.

Example: Tell when a block is inside or outside a box.

Standard 5

Measurement

Students understand the concept of time and units to measure it. They understand that objects have length, capacity, weight, and temperature, and that they can compare objects using these qualities.

K.5.1 Make direct comparisons of the length, capacity, weight, and temperature of objects and recognize which object is shorter, longer, taller, lighter, heavier, warmer, cooler or holds more.

Example: Hold two books side by side to see which is shorter. Hold one in each hand to see which is heavier.

K.5.2 Understand concepts of time: morning, afternoon, evening, today, yesterday, tomorrow, week, month, and year. Understand that clocks and calendars are tools that measure time.

Example: Use a calendar to find the number of days in the month of your birthday.

Standard 6

Problem Solving

Students make decisions about how to set up a problem.

K.6.1 Choose the approach, materials, and strategies to use in solving problems.

Example: Solve the problem: "There are four blocks on the table and a box of blocks that is closed. The teacher says that there are five blocks in the box. Find the number of blocks in all, without opening the box." Decide to draw a picture.

K.6.2 Use tools such as objects or drawings to model problems.

Example: In the first example, draw a picture of the four blocks that you can see, and then draw five more blocks for the ones that you cannot see.

Students solve problems in reasonable ways and justify their reasoning.

K.6.3 Explain the reasoning used with concrete objects and pictures.

Example: In the first example, count the number of blocks that you have drawn and write the number that represents the total.

K.6.4 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, open the box of blocks and place them on the table. Count the total number of blocks on the table to see whether your drawing was correct.

NOTES

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 1:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students develop this understanding by first counting sets of objects and then moving on to writing numbers in figures. They learn how we group numbers in tens and ones, allowing them to write numbers up to 100. They find the number one more or one less than a given number. They put numbers up to 10 in order of size and use the terms *first*, *second*, *third*, etc. Students also learn about fractions, understanding that fractions compare a part of a set to the whole set.

Standard 2 — Computation

Fluency in computation is essential. As students learn about the whole numbers up to 100, they also learn how to add and subtract them. They use objects to join sets together (for addition) and to remove objects from sets (for subtraction). They become familiar with different ways of looking at the same number using objects and figures. They also learn that addition and subtraction are opposites of each other and that zero has special properties.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level relate word problems to number sentences in symbols, such as $7 + 6 = 13$, and learn some of the rules relating addition and subtraction. They also continue number patterns using addition.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They describe and draw simple shapes, comparing and sorting them by such attributes as size and number of sides. They learn the meaning of words, like *near* and *behind*, that relate to positions in space and use them to give and follow directions. They identify objects as two- or three-dimensional and describe the faces of solid objects. They also recognize geometric shapes in the world around them.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students begin their study of measurement by comparing objects' length, weight, temperature, etc. Then they become more precise and find, for example, that the length of their desk is 8 pencil-lengths. From this, they move toward understanding the need for standard units of length: inch, foot, yard, centimeter, and meter. They learn how to tell the time on a clock to the nearest half hour. They also learn about money: the values of pennies, nickels, and dimes.

Standard 6 — Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with numbers, geometry, or measurement, for example, students at this level move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, $\frac{1}{4}$, one divided by four, 0.25, $\frac{1}{8} + \frac{1}{8}$, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: $+$, $=$, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense

Students understand symbols, objects, and pictures used to represent numbers up to 100 and show an understanding of fractions.

1.1.1 Count, read, and write whole numbers* up to 100.

Example: Read "seventy-two" for the number 72.

1.1.2 Count and group objects in ones and tens.

Example: Separate a group of 34 blocks into three groups of 10 blocks and 4 single blocks.

1.1.3 Identify the number of tens and ones in numbers less than 100.

Example: How many tens and how many ones are in 56? Explain your answer.

1.1.4 Name the number that is one more than or one less than any number up to 100.

Example: Name the number one less than 78.

1.1.5 Compare whole numbers up to 10 and arrange them in numerical order.

Example: Arrange the numbers 5, 2, and 9 in order from greatest to least.

1.1.6 Match the number names (*first*, *second*, *third*, etc.) with an ordered set of up to 10 items.

Example: Point out the fifth child from the front of a line of children.

1.1.7 Recognize when a shape is divided into congruent (matching) parts.

Example: Given a rectangle with lines dividing it into parts, decide whether the parts are the same size.

1.1.8 For a shape divided into 8 or fewer congruent (matching) parts, describe a shaded portion as "__ out of __ parts" and write the fraction.

Example: Given a circle divided into 4 equal parts with 3 of the parts shaded, describe the shaded portion as "3 out of 4 parts" and write the fraction for the shaded portion.

1.1.9 For a set of 8 or fewer objects, describe a subset as "__ out of __ parts" and write the fraction.

Example: Given 3 red pencils and 2 blue pencils, describe the subset of red pencils as "3 out of 5 parts" and write the fraction of the pencils that are red.

1.1.10 Represent, compare, and interpret data using pictures and picture graphs.

Example: Use a picture graph to show how many dogs, cats, etc. your friends have. Which kind of pet appears most often? Explain your answer.

* whole number: 0, 1, 2, 3, etc.

Standard 2

Computation

Students demonstrate the meaning of addition and subtraction and use these operations to solve problems.

1.2.1 Show the meaning of addition (putting together, increasing) using objects.

Example: Put together 3 pencils and 5 pencils. Tell how many pencils you have and explain what you are doing.

1.2.2 Show the meaning of subtraction (taking away, comparing, finding the difference) using objects.

Example: Take away 6 blocks from a group of 10. Tell how many blocks are left and explain what you are doing.

1.2.3 Show equivalent forms of the same number (up to 20) using objects, diagrams, and numbers.

Example: Write 15 as $8 + 7$, $5 + 5 + 5$, $10 + 5$, $15 + 0$, $17 - 2$, etc.

1.2.4 Demonstrate mastery of the addition facts (for totals up to 20) and the corresponding subtraction facts.

Example: Add $11 + 8$, subtract $16 - 9$, add $4 + 7$.

1.2.5 Understand the meaning of the symbols $+$, $-$, and $=$.

Example: Use symbols to write the number sentence "one added to three equals four."

1.2.6 Understand the role of zero in addition and subtraction.

Example: You start with 6 eggs and then give away 0 eggs. How many eggs do you have now?

1.2.7 Understand and use the inverse relationship between addition and subtraction facts (such as $4 + 2 = 6$, $6 - 2 = 4$, etc.) to solve simple problems.

Example: List three other facts using addition or subtraction that are related to $3 + 5 = 8$.

Standard 3

Algebra and Functions

Students use number sentences with the symbols $+$, $-$, and $=$ to solve problems.

1.3.1 Write and solve number sentences from problem situations involving addition and subtraction.

Example: You have 3 pencils and your friend has 2 pencils. You want to know how many pencils you have altogether. Write a number sentence for this problem and use it to find the total number of pencils.

1.3.2 Create word problems that match given number sentences involving addition and subtraction.

Example: Tell a story or draw a picture for a problem that can be solved using the number sentence $3 + 6 = 9$.

1.3.3 Recognize and use the relationship between addition and subtraction.

Example: Start with 8 blocks. Add 5 more blocks. How many do you have? Now take away 5 blocks. How many do you have now? Explain your answer.

1.3.4 Create and extend number patterns using addition.

Example: A number pattern begins with these numbers: 1, 3, 5, …. Tell what the next number will be and explain how you decided on that number.

Standard 4

Geometry

Students identify common geometric shapes, classify them by common attributes, and describe their relative position or their location in space.

1.4.1 Identify, describe, compare, sort, and draw triangles, rectangles, squares, and circles.

Example: Draw a square and a circle and write their names next to them.

1.4.2 Identify triangles, rectangles, squares, and circles as the faces* of three-dimensional objects.

Example: Look at a collection of solid objects and find triangles and squares on their sides.

1.4.3 Classify and sort familiar plane and solid objects by position, shape, size, roundness, and other attributes. Explain the rule used.

Example: Group a collection of objects by something they have in common. Explain your grouping.

1.4.4 Identify objects as two-dimensional or three-dimensional.

Example: Sort various objects (cube, square, triangle, prism) into the categories "two-dimensional" and "three-dimensional." Explain your choices.

1.4.5 Give and follow directions for finding a place or object.

Example: Show someone how to get to the school library by making a map or diagram.

1.4.6 Arrange and describe objects in space by position and direction: near, far, under, over, up, down, behind, in front of, next to, to the left or right of.

Example: Name objects that are near your desk and objects that are in front of it. Explain why there may be some objects in both groups.

1.4.7 Identify geometric shapes and structures in the environment and specify their location.

Example: Find as many rectangles as you can in your classroom. Record the rectangles that you found by making drawings or using a camera.

* face: a flat side, like the front of a cereal box

Standard 5

Measurement

Students learn how to measure length, as well as how to compare, order, and describe other kinds of measurement.

1

1.5.1 Measure the length of objects by repeating a nonstandard unit or a standard unit.

Example: Measure the length of your desk in pencil-lengths.

1.5.2 Use different units to measure the length of the same object and predict whether the measure will be greater or smaller when a different unit is used.

Example: If you measure your desk with a shorter pencil, will the number of pencil-lengths be more or less? Measure the desk to find out your answer.

1.5.3 Recognize the need for a fixed unit of length.

Example: Give students different lengths of string and have them measure the width of a doorway. Talk about why their answers are different and the kinds of problems this can cause.

1.5.4 Measure and estimate the length of an object to the nearest inch and centimeter.

Example: Have some students measure the width of the doorway in inches and some measure it in centimeters. Discuss why these are better ways of measuring than using the pieces of string.

1.5.5 Compare and order objects according to area, capacity, weight, and temperature, using direct comparison or a nonstandard unit.

Example: Use a scale or balance to see how many crayons weigh the same as a shoe.

1.5.6 Tell time to the nearest half-hour and relate time to events (before/after, shorter/longer).

Example: Is recess before or after lunch?

1.5.7 Identify and give the values of collections of pennies, nickels, and dimes.

Example: How many pennies have the same value as two nickels?

Standard 6

Problem Solving

Students make decisions about how to set up a problem.

1.6.1 Choose the approach, materials, and strategies to use in solving problems.

Example: Solve the problem: "The number 10 can be written in different ways using addition: $10 = 4 + 6$ or $10 = 1 + 9$ … . Find how many ways you can write 10 by adding two numbers." Use blocks to set up the problem.

1.6.2 Use tools such as objects or drawings to model problems.

Example: In the first example, show the number 10 using addition of whole numbers by counting out ten blocks. Divide them into two piles and write a number sentence that shows the number in each pile of blocks.

Students solve problems and justify their reasoning.

1.6.3 Explain the reasoning used and justify the procedures selected in solving a problem.

Example: In the first example, make two piles of ten blocks; separate one block from the first pile and count the number of blocks left. Separate two blocks from the second pile and count the number left. Describe any pattern of numbers that you find.

1.6.4 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, check your results by setting out 10 blocks showing $1 + 9$, another 10 blocks showing $2 + 8$, and so on. Continue to count out piles of 10 blocks to find the total number of ways that ten blocks can be separated into two piles. Describe the patterns that you find and how you know that you have found all of them.

1.6.5 Understand and use connections between two problems.

Example: Use the problem you have just solved to find how many ways you can write 16 by adding two numbers.

NOTES

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 2:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students develop this understanding by first using sets of objects and then moving on to writing numbers in figures. They learn how we group numbers in tens and ones, which allows them to write numbers up to 100. They count by ones, twos, fives, and tens. They find the number ten more or ten less than a given number. They identify odd and even numbers and put numbers in order of size. They use the terms first, second, third, etc. Students also extend their knowledge of fractions, including learning how to compare the sizes of simple fractions.

Standard 2 — Computation

Fluency in computation is essential. As students learn about the whole numbers up to 100, they also learn how to add and subtract them. They use objects to join sets together (for addition) and to remove objects from sets (for subtraction). They also learn to add and subtract with figures using mental arithmetic.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level make simple patterns with numbers and continue these number patterns using addition and subtraction. They also relate word problems to number sentences such as $28 - 15 = 13$ and use rules for addition to check results.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They identify and describe simple shapes, such as circles, triangles, squares, rectangles, and cubes. Students construct simple two- and three-dimensional shapes and describe and sort them using their faces, edges, and corners. They identify shapes that are congruent (i.e., the same shape and size). They also investigate how shapes are made from other shapes and recognize geometric shapes in the world around them.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students measure in order to compare objects' lengths, areas, weights, temperatures, etc. They learn why we use standard units of length (inch, foot, yard, centimeter, and meter) and measure objects using these units. In a similar way, they learn how to measure weight, capacity, and temperature in standard units. They also learn about time (hours in a day, months in a year, etc.) and how to tell the time on a clock to the nearest five minutes. They learn about money: the values of the coins and the value of a collection of coins and dollars.

Standard 6 — Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with numbers, geometry, or measurement, for example, students move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

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Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

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The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, $\frac{1}{4}$, one divided by four, 0.25, $\frac{1}{8} + \frac{1}{8}$, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: $+$, $=$, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense

Students understand the relationships among numbers, quantities, and place value in whole numbers up to 100. They understand that fractions may refer to parts of a set* and parts of a whole.*

2.1.1 Count by ones, twos, fives, and tens to 100.

Example: Count 74 pencils by groups of tens and twos.

2.1.2 Identify the pattern of numbers in each group of ten, from tens through nineties.

Example: What pattern do you see on a hundreds chart for the numbers 12, 22, 32, etc.?

2.1.3 Identify numbers up to 100 in various combinations of tens and ones.

Example: 32 = 3 tens + 2 ones = 2 tens + 12 ones, etc.

2.1.4 Name the number that is ten more or ten less than any number 10 through 90.

Example: Name the number ten more than 54.

2.1.5 Compare whole numbers up to 100 and arrange them in numerical order.

Example: Put the numbers in order of size: 95, 28, 42, 31.

2.1.6 Match the number names (*first*, *second*, *third*, etc.) with an ordered set of up to 100 items.

Example: Identify the seventeenth letter of the alphabet.

2.1.7 Identify odd and even numbers up to 100.

Example: Find the odd numbers in this set: 44, 31, 100, 57, 28.

2.1.8 Recognize fractions as parts of a whole or parts of a group (up to 12 parts).

Example: Divide a cardboard rectangle into 8 equal pieces. Shade 5 pieces and write the fraction for the shaded part.

2.1.9 Recognize, name, and compare the unit fractions: ½, ⅓, ¼, ⅕, ⅙, ⅛, 1/10, and 1/12.

Example: Which is larger, ⅓ or ⅙? Explain your answer.

2.1.10 Know that, when all fractional parts are included, the result is equal to the whole and to one.

Example: What is another way of saying six sixths? Explain your answer.

2.1.11 Collect and record numerical data in systematic ways.

Example: Measure the hand span in whole centimeters of each student in your class. Keep a record of the answers they give you.

2.1.12 Represent, compare, and interpret data using tables, tally charts, and bar graphs.

Example: Make a tally of your classmates' favorite colors and draw a bar graph. Name the color that is most popular and the color that is the favorite of the fewest people.

* whole number: 0, 1, 2, 3, etc.

* set: collection of objects, numbers, etc.

Standard 2

Computation

Students solve simple problems involving addition and subtraction of numbers up to 100.

2.2.1 Model addition of numbers less than 100 with objects and pictures.

Example: Use blocks to find the sum of 26 and 15.

2.2.2 Add two whole numbers less than 100 with and without regrouping.

Example: $36 + 45 = ?$.

2.2.3 Subtract two whole numbers less than 100 without regrouping.

Example: $86 - 55 = ?$.

2.2.4 Understand and use the inverse relationship between addition and subtraction.

Example: Understand that $89 - 17 = 72$ means that $72 + 17 = 89$.

2.2.5 Use estimation to decide whether answers are reasonable in addition problems.

Example: Your friend says that $13 + 24 = 57$. Without solving, explain why you think the answer is wrong.

2.2.6 Use mental arithmetic to add or subtract 0, 1, 2, 3, 4, 5, or 10 with numbers less than 100.

Example: In a game, Mia and Noah are making addition problems. They make two two-digit numbers out of the four given numbers 1, 2, 3, and 4. Each number is used exactly once. The winner is the one who makes two numbers whose sum is the largest. Mia had 24 and 31; Noah had 21 and 43. Who won the game? How do you know? Show a way to beat both of them.

Standard 3

Algebra and Functions

Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction.

2.3.1 Relate problem situations to number sentences involving addition and subtraction.

Example: You have 13 pencils and your friend has 12 pencils. You want to know how many pencils you have altogether. Write a number sentence for this problem and use it to find the total number of pencils.

2.3.2 Use the commutative* and associative* properties for addition to simplify mental calculations and to check results.

Example: Add the numbers 5, 17, and 13 in this order. Now add them in the order 17, 13, and 5. Which was easier? Why?

2.3.3 Recognize and extend a linear pattern by its rules.

Example: One horse has 4 legs, two horses have 8 legs, and so on. Continue the pattern to find how many legs five horses have.

2.3.4 Create, describe, and extend number patterns using addition and subtraction.

Example: What is the next number: 23, 21, 19, 17, …? How did you find your answer?

* commutative property: the order when adding or multiplying numbers makes no difference (e.g., $5 + 3 = 3 + 5$), but note that this is not true for subtraction or division

* associative property: the grouping when adding or multiplying numbers makes no difference (e.g., in $5 + 3 + 2$, adding 5 and 3 and then adding 2 is the same as 5 added to $3 + 2$), but note that this is not true for subtraction or division

Standard 4

Geometry

Students identify and describe the attributes of common shapes in the plane and of common objects in space.

2.4.1 Construct squares, rectangles, triangles, cubes, and rectangular prisms* with appropriate materials.

Example: Use blocks to make a rectangular prism.

2.4.2 Describe, classify, and sort plane and solid geometric shapes (triangle, square, rectangle, cube, rectangular prism) according to the number and shape of faces* and the number of sides, edges, and/or vertices*.

Example: How many vertices does a cube have?

2.4.3 Investigate and predict the result of putting together and taking apart two-dimensional and three-dimensional shapes.

Example: Use objects or a drawing program to find other shapes that can be made from a rectangle and a triangle. Use sketches or a drawing program to show several ways that a rectangle can be divided into three triangles.

2.4.4 **Identify congruent* two-dimensional shapes in any position.**

Example: In a collection of rectangles, pick out those that are the same shape and size.

2.4.5 **Recognize geometric shapes and structures in the environment and specify their locations.**

Example: Look for combinations of shapes in the buildings around you.

* rectangular prism: a box with six rectangles for sides, like a cereal box
* face: a flat side, like the front of the cereal box
* vertices: corners (vertex: corner)
* congruent: the term to describe two figures that are the same shape and size

Standard 5

Measurement

Students understand how to measure length, temperature, capacity, weight, and time in standard units.

2.5.1 **Measure and estimate length to the nearest inch, foot, yard, centimeter, and meter.**

Example: Measure the length of your classroom to the nearest foot.

2.5.2 **Describe the relationships among inch, foot, and yard. Describe the relationship between centimeter and meter.**

Example: How many inches are in a yard?

2.5.3 **Decide which unit of length is most appropriate in a given situation.**

Example: Would you use yards or inches to measure the length of your school books? Explain your answer.

2.5.4 **Estimate area and use a given object to measure the area of other objects.**

Example: Make a class estimate of the number of sheets of notebook paper that would be needed to cover the classroom door. Then use measurements to compute the area of the door.

2.5.5 **Estimate and measure capacity using cups and pints.**

Example: Make a reasonable estimate of the number of pints a juice pitcher holds.

2.5.6 **Estimate weight and use a given object to measure the weight of other objects.**

Example: About how many jellybeans will you need to put on one side of a balance scale to balance with a box of chalk? Count out the number of jellybeans that you guessed would be needed and see whether your estimate was close. Explain the results of your estimation and weighing.

2.5.7 **Recognize the need for a fixed unit of weight.**

Example: Estimate the number of paperclips needed to balance with a box of chalk. Will it be the same as the number of jellybeans? Explain your answer.

2.5.8 Estimate temperature. Read a thermometer in Celsius and Fahrenheit.

Example: What do you think the temperature is today? Look at the thermometer to check.

2.5.9 Tell time to the nearest quarter hour, be able to tell five-minute intervals, and know the difference between a.m. and p.m.

Example: When does your favorite TV program start?

2.5.10 Know relationships of time: seconds in a minute; minutes in an hour; hours in a day; days in a week; and days, weeks, and months in a year.

Example: How many days are in a year?

2.5.11 Find the duration of intervals of time in hours.

Example: Your trip began at 9:00 a.m. and ended at 3:00 p.m. How long were you traveling?

2.5.12 Find the value of a collection of pennies, nickels, dimes, quarters, half-dollars, and dollars.

Example: You have 3 pennies, 4 nickels, and 2 dimes. How much money do you have? Explain your answer.

Standard 6

Problem Solving

Students make decisions about how to set up a problem.

2.6.1 Choose the approach, materials, and strategies to use in solving problems.

Example: Solve the problem: "Count the number of squares on the surface of a cube. Put two cubes together and count the number of visible squares. Repeat this step with 3, 4, 5, …, cubes in a line. Find a rule for the number of squares." Use blocks to set up the problem.

2.6.2 Use tools such as objects or drawings to model problems.

Example: In the first example, place blocks together. Each time you add a block, count the number of squares and record it.

Students solve problems and justify their reasoning.

2.6.3 Explain the reasoning used and justify the procedures selected in solving a problem.

Example: In the first example, notice that the number goes up by 4 each time a block is added. Observe that, as you add each cube, you gain 6 squares but lose 2 where the blocks are joined.

2.6.4 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, check your results by setting out 10 blocks and counting the number of squares on each long side and then the two at the ends. See how this fits with your rule of adding 4 each time.

2.6.5 Understand and use connections between two problems.

Example: Use the method of the problem you have just solved to find what happens when the cubes are not all in a line.

NOTES

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The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 3:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students extend their understanding of the place value system to count, read, and write numbers up to 1,000. They learn to order and round numbers up to 1,000. They develop the concept of equivalent fractions — fractions that look different, but have the same value — and use their understanding of equivalent fractions to compare the sizes of fractions. They also begin to develop the concept of decimals as a different way of representing fractional numbers.

Standard 2 — Computation

Fluency in computation is essential. As students learn about the whole numbers up to 1,000, they learn how to add and subtract them. They develop the concepts of multiplication and division from addition and subtraction and learn basic multiplication and division facts. They also start to add and subtract fractions with the same denominator.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level represent relationships with numeric equations and use those equations to solve problems. They continue number patterns involving multiplication and use some of the rules for multiplication to check results. They begin to develop the concept of a function and the relationship between numbers and number lines.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They identify quadrilaterals and learn about right angles as a basis for comparing other angles. They describe and classify three-dimensional shapes. They use the basic terms point, line, and line segment to describe shapes. They also develop the concept of mirror-image symmetry and draw lines of symmetry.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students measure length to the nearest half-inch, add units of length, and find the perimeters of shapes. They estimate area and volume in preparation for developing formulas for calculating them. They estimate, measure, and compare weights, capacities, and temperatures in standard units. They also learn about money: the value of any collection of coins and dollars, writing money using the $ symbol, and deciding whether they have enough money to make a purchase.

Standard 6 — Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with numbers, geometry, or measurement, for example, students move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

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The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense

Students understand the relationships among numbers, quantities, and place value in whole numbers up to 1,000. They understand the relationship among whole numbers, simple fractions, and decimals.*

3.1.1 Count, read, and write whole numbers up to 1,000.

Example: Write 349 for the number "three hundred forty-nine."

3.1.2 Identify and interpret place value in whole numbers up to 1,000.

Example: Understand that the 7 in 479 represents 7 tens or 70.

3.1.3 Use words, models, and expanded form to represent numbers up to 1,000.

Example: Recognize that $492 = 400 + 90 + 2$.

3.1.4 Identify any number up to 1,000 in various combinations of hundreds, tens, and ones.

Example: 325 can be written as 3 hundreds, 2 tens, and 5 ones, or as 2 hundreds, 12 tens, and 5 ones, etc.

3.1.5 Compare whole numbers up to 1,000 and arrange them in numerical order.

Example: What is the smallest whole number you can make using the digits 4, 9, and 1? Use each digit exactly once.

3.1.6 Round numbers less than 1,000 to the nearest ten and the nearest hundred.

Example: Round 548 to the nearest ten.

3.1.7 Identify odd and even numbers up to 1,000 and describe their characteristics.

Example: Find the even number: 47, 106, 357, 629.

3.1.8 Show equivalent fractions* using equal parts.

Example: Draw pictures to show that $\frac{3}{5}$, $\frac{6}{10}$, and $\frac{9}{15}$ are equivalent fractions.

3.1.9 Identify and use correct names for numerators and denominators.

Example: In the fraction $\frac{3}{5}$, name the numerator and denominator.

3.1.10 Given a pair of fractions, decide which is larger or smaller by using objects or pictures.

Example: Is $\frac{3}{4}$ of a medium pizza larger or smaller than $\frac{1}{2}$ of a medium pizza? Explain your answer.

3.1.11 Given a set* of objects or a picture, name and write a decimal to represent tenths and hundredths.

Example: You have a pile of 100 beans and 72 of them are lima beans. Write the decimal that represents lima beans as a part of the whole pile of beans.

3.1.12 Given a decimal for tenths, show it as a fraction using a place-value model.

Example: Shade the part of a square that represents 0.7 and write the number $\frac{7}{10}$.

3.1.13 Interpret data displayed in a circle graph and answer questions about the situation.

Example: Have the students in your class choose the pizza they like best from these choices: cheese, sausage, pepperoni. Use a spreadsheet to enter the number of students who chose each kind and make a circle graph of the data. Determine the most popular and the least popular kind of pizza, and explain what the circle and each pie slice represent.

3.1.14 Identify whether everyday events are certain, likely, unlikely, or impossible.

Example: It is raining in your neighborhood. Is it certain, likely, unlikely, or impossible that the tree in your front yard will get wet?

3.1.15 Record the possible outcomes for a simple probability experiment.

Example: Have a partner toss a coin while you keep a tally of the outcomes. Exchange places with your partner and repeat the experiment. Explain your results to the class.

* whole number: 0, 1, 2, 3, etc.
* equivalent fractions: fractions with the same value (e.g., $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, etc.)
* set: collection of objects, numbers, etc.

Standard 2

Computation

Students solve problems involving addition and subtraction of whole numbers. They model and solve simple problems involving multiplication and division.

3.2.1 Add and subtract whole numbers up to 1,000 with or without regrouping, using relevant properties of the number system.

Example: $854 - 427 = ?$. Explain your method.

3.2.2 Represent the concept of multiplication as repeated addition.

Example: Lynn made 3 baskets each week for 4 weeks. Draw a picture to show how many baskets she made.

3.2.3 Represent the concept of division as repeated subtraction, equal sharing, and forming equal groups.

Example: Bob shared 10 cookies among 5 friends. Draw a picture to show how many cookies each friend got.

3.2.4 Know and use the inverse relationship between multiplication and division facts, such as $6 \times 7 = 42$, $42 \div 7 = 6$, $7 \times 6 = 42$, $42 \div 6 = 7$.

Example: Find other facts related to $8 \times 3 = 24$.

3.2.5 Show mastery of multiplication facts for 2, 5, and 10.

Example: Know the answer to 6×5.

3.2.6 Add and subtract simple fractions with the same denominator.

Example: Add $\frac{3}{8}$ and $\frac{1}{8}$. Explain your answer.

3.2.7 Use estimation to decide whether answers are reasonable in addition and subtraction problems.

Example: Your friend says that $79 - 22 = 27$. Without solving, explain why you think the answer is wrong.

3.2.8 Use mental arithmetic to add or subtract with numbers less than 100.

Example: Subtract 35 from 86 without using pencil and paper.

Standard 3

Algebra and Functions

Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number and functional relationships.

3.3.1 Represent relationships of quantities in the form of a numeric expression or equation.

Example: Bill's mother gave him money to buy three drinks that cost 45 cents each at the concession stand. When he returned to the bleachers, he gave 25 cents change to his mother. Write an equation to find the amount of money Bill's mother originally gave him.

3.3.2 Solve problems involving numeric equations.

Example: Use your equation from the last example to find the amount of money that Bill's mother gave him, and justify your answer.

3.3.3 Choose appropriate symbols for operations and relations to make a number sentence true.

Example: What symbol is needed to make the number sentence 4 __ 3 = 12 true?

3.3.4 Understand and use the commutative* and associative* properties of multiplication.

Example: Multiply the numbers 7, 2, and 5 in this order. Now multiply them in the order 2, 5, and 7. Which was easier? Why?

3.3.5 Create, describe, and extend number patterns using multiplication.

Example: What is the next number: 3, 6, 12, 24, …? How did you find your answer?

3.3.6 Solve simple problems involving a functional relationship between two quantities.

Example: Ice cream sandwiches cost 20 cents each. Find the costs of 1, 2, 3, 4, … ice cream sandwiches. What pattern do you notice? Continue the pattern to find the cost of enough ice cream sandwiches for the class.

3.3.7 Plot and label whole numbers on a number line up to 10.

Example: Mark the position of 7 on a number line up to 10.

* commutative property: the order when adding or multiplying numbers makes no difference (e.g., 5 + 3 = 3 + 5), but note that this rule is not true for subtraction or division

* associative property: the grouping when adding or multiplying numbers makes no difference (e.g., in 5 + 3 + 2, adding 5 and 3 and then adding 2 is the same as 5 added to 3 + 2), but note that this rule is not true for subtraction or division

Standard 4

Geometry

Students describe and compare the attributes of plane and solid geometric shapes and use their understanding to show relationships and solve problems.

3.4.1 Identify quadrilaterals* as four-sided shapes.

Example: Which of these are quadrilaterals: square, triangle, rectangle?

3.4.2 Identify right angles in shapes and objects and decide whether other angles are greater or less than a right angle.

Example: Identify right angles in your classroom. Open the classroom door until it makes a right angle with one wall and explain what you are doing.

3.4.3 Identify, describe, and classify: cube, sphere*, prism*, pyramid, cone, and cylinder.

Example: Describe the faces of a pyramid and identify its characteristics.

3.4.4 Identify common solid objects that are the parts needed to make a more complex solid object.

Example: Describe and draw a house made from a prism and a pyramid.

3.4.5 Draw a shape that is congruent* to another shape.

Example: Draw a triangle that is congruent to a given triangle. You may use a ruler and pencil or the drawing program on a computer.

3.4.6 Use the terms *point*, *line*, and *line segment* in describing two-dimensional shapes.

Example: Describe the way a triangle is made of points and line segments and how you know it is a triangle.

3.4.7 Draw line segments and lines.

Example: Draw a line segment three inches long.

3.4.8 Identify and draw lines of symmetry in geometric shapes (by hand or using technology).

Example: Use pencil and paper or a drawing program to draw lines of symmetry in a square. Discuss your findings.

3.4.9 Sketch the mirror image reflections of shapes.

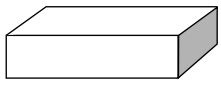
Example: Hold up a cardboard letter F to a mirror. Draw the letter and the shape you see in the mirror.

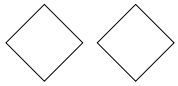
3.4.10 Recognize geometric shapes and their properties in the environment and specify their locations.

Example: Write the letters of the alphabet and draw all the lines of symmetry that you see.

* quadrilateral: a two-dimensional figure with four sides 

* sphere: a shape best described as that of a round ball, such as a baseball, that looks the same when seen from all directions.

* prism: a solid shape with fixed cross-section (a right prism is a solid shape with two parallel faces that are congruent polygons and other faces that are rectangles) 

* congruent: the term to describe two figures that are the same shape and size 

Standard 5

Measurement

Students choose and use appropriate units and measurement tools for length, capacity, weight, temperature, time, and money.

3.5.1 Measure line segments to the nearest half-inch.

Example: Measure the length of a side of a triangle.

3.5.2 Add units of length that may require regrouping of inches to feet or centimeters to meters.

Example: Add the lengths of three sheets of paper. Give your answer in feet and inches.

3.5.3 Find the perimeter of a polygon*.

Example: Find the perimeter of a table in centimeters. Explain your method.

3.5.4 Estimate or find the area of shapes by covering them with squares.

Example: How many square tiles do we need to cover this desk?

3.5.5 Estimate or find the volumes of objects by counting the number of cubes that would fill them.

Example: How many of these cubes will fill the box?

3.5.6 Estimate and measure capacity using quarts, gallons, and liters.

Example: This bottle holds one liter. Estimate how many liters the sink holds.

3.5.7 Estimate and measure weight using pounds and kilograms.

Example: Estimate the weight of your book bag in pounds.

3.5.8 Compare temperatures in Celsius and Fahrenheit.

Example: Measure the room temperature using a thermometer that has both Celsius and Fahrenheit units. If the temperature in the room measures 70°F, will the Celsius measurement be higher or lower?

3.5.9 Tell time to the nearest minute and find how much time has elapsed.

Example: You start a project at 9:10 a.m. and finish the project at 9:42 a.m. How much time has passed?

3.5.10 Find the value of any collection of coins and bills. Write amounts less than a dollar using the ¢ symbol and write larger amounts in decimal notation using the $ symbol.

Example: You have 5 quarters and 2 dollar bills. How much money is that? Write the amount.

3.5.11 Use play or real money to decide whether there is enough money to make a purchase.

Example: You have $5. Can you buy two books that cost $2.15 each? What about three books that cost $1.70 each? Explain how you know.

3.5.12 Carry out simple unit conversions within a measurement system (e.g., centimeters to meters, hours to minutes).

Example: How many minutes are in 3 hours?

* polygon: a two-dimensional shape with straight sides (e.g., triangle, rectangle, pentagon)

Standard 6

Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

3.6.1 Analyze problems by identifying relationships, telling relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

Example: Solve the problem: "Start with any number. If it is even, halve it. If it is odd, add 1. Do the same with the result and keep doing that. Find what happens by trying different numbers." Try two or three numbers and look for patterns.

3.6.2 Decide when and how to break a problem into simpler parts.

Example: In the first example, find what happens to all the numbers up to 10.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

3.6.3 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, use your results for the numbers up to 10 to find what happens to all the numbers up to 20.

3.6.4 Express solutions clearly and logically by using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, explain what happens to all the numbers that you tried.

3.6.5 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: Measure the length and width of a room to the nearest meter to find how many student desks will fit in it. Would this be an accurate enough method if you were carpeting the room?

3.6.6 Know and use strategies for estimating results of whole-number addition and subtraction.

Example: You buy 2 bags of candy for $1.05 each. The cashier tells you that will be $1.70. Does that surprise you? Why or why not?

3.6.7 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, notice that the result of adding 1 to an odd number is always even. Use this to check your calculations.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

3.6.8 Decide whether a solution is reasonable in the context of the original situation.

Example: In the example about fitting desks into a room, would an answer of 1,000 surprise you?

3.6.9 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: Change the first example so that you multiply odd numbers by 2 or 3 or 4 or 5, before adding 1. Describe the pattern you see.

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 4:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students extend their understanding of the place value system to count, read, and write whole numbers up to 1,000,000 and decimals to two places. They order and compare whole numbers using the correct symbols for greater than and less than. They extend the concept of fractions to mixed numbers, learning how fractions are related to whole numbers. They also extend their skills with decimals and how they relate to fractions.

Standard 2 — Computation

Fluency in computation is essential. As students learn about numbers, they also learn how to add, subtract, multiply, and divide them. They understand the special roles of 0 and 1 in multiplication and division. They also add and subtract fractions and decimals, learning how these different representations of numbers can be manipulated.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level develop an understanding of the fundamental concept of a variable — having a letter represent all numbers of a certain kind. They use this to write formulas and equations, including equations that give the rule for a function. They continue number patterns involving multiplication and division. They recognize and apply the relationships among the four operations of addition, subtraction, multiplication, and division. They further develop the connection between numbers and number lines, including estimating positions on a number line.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They identify, describe, and draw such concepts as acute angles and parallel lines. They describe shapes and objects, including special quadrilaterals such as rhombuses and trapezoids. They identify congruent quadrilaterals and explain their reasoning using specific geometric terms. They draw lines of symmetry for various polygons, and they construct cubes and prisms, developing their ability to work in three dimensions.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students measure length to the nearest eighth-inch and millimeter and subtract units of length. They develop and use the formulas for calculating perimeters and areas of rectangles. They compare the concepts of volume and capacity. They add time intervals and calculate the amount of change from a purchase.

Standard 6 — Data Analysis and Probability

Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they represent data on a number line and in frequency tables, interpret data graphs to answer questions, and summarize the results of probability experiments in an organized way.

Standard 7 — Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with numbers, geometry, or measurement, for example, students move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Number Sense

Students understand the place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions.*

4.1.1 Read and write whole numbers up to 1,000,000.

Example: Read aloud the number 394,734.

4.1.2 Identify and write whole numbers up to 1,000,000, given a place-value model.

Example: Write the number that has 2 hundred thousands, 7 ten thousands, 4 thousands, 8 hundreds, 6 tens, and 2 ones.

4.1.3 Round whole numbers up to 10,000 to the nearest ten, hundred, and thousand.

Example: Is 7,683 closer to 7,600 or 7,700? Explain your answer.

4.1.4 Order and compare whole numbers using symbols for "less than" (<), "equal to" (=), and "greater than" (>).

Example: Put the correct symbol in 328 __ 142.

4.1.5 Rename and rewrite whole numbers as fractions.

Example: $3 = {}^{6}/_{2} = {}^{9}/_{3} = {}^{?}/_{4} = {}^{?}/_{5}$.

4.1.6 Name and write mixed numbers, using objects or pictures.

Example: You have 5 whole straws and half a straw. Write the number that represents these objects.

4.1.7 Name and write mixed numbers as improper fractions, using objects or pictures.

Example: Use a picture of 3 rectangles, each divided into 5 equal pieces, to write $2{}^{3}/_{5}$ as an improper fraction.

4.1.8 Write tenths and hundredths in decimal and fraction notations. Know the fraction and decimal equivalents for halves and fourths (e.g., $1/2 = 0.5 = 0.50$, $7/4 = 1{}^{3}/_{4} = 1.75$).

Example: Write ${}^{26}/_{100}$ and $2{}^{3}/_{4}$ as decimals.

4.1.9 Round two-place decimals to tenths or to the nearest whole number.

Example: You ran the 50-yard dash in 6.73 seconds. Round your time to the nearest tenth.

* whole number: 0, 1, 2, 3, etc.

Standard 2

Computation

Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among these operations. They extend their use and understanding of whole numbers to the addition and subtraction of simple fractions and decimals.

4.2.1 Understand and use standard algorithms* for addition and subtraction.

Example: 45,329 + 6,984 = ?, 36,296 − 12,075 = ?.

4.2.2 Represent as multiplication any situation involving repeated addition.

Example: Each of the 20 students in your physical education class has 3 tennis balls. Find the total number of tennis balls in the class.

4.2.3 Represent as division any situation involving the sharing of objects or the number of groups of shared objects.

Example: Divide 12 cookies equally among 4 students. Divide 12 cookies equally to find out how many people can get 4 cookies. Compare your answers and methods.

4.2.4 Demonstrate mastery of the multiplication tables for numbers between 1 and 10 and of the corresponding division facts.

Example: Know the answers to 9 × 4 and 35 ÷ 7.

4.2.5 Use a standard algorithm to multiply numbers up to 100 by numbers up to 10, using relevant properties of the number system.

Example: 67 × 3 = ?.

4.2.6 Use a standard algorithm to divide numbers up to 100 by numbers up to 10 without remainders, using relevant properties of the number system.

Example: 69 ÷ 3 = ?.

4.2.7 Understand the special properties of 0 and 1 in multiplication and division.

Example: Know that 73 × 0 = 0 and that 42 ÷ 1 = 42.

4.2.8 Add and subtract simple fractions with different denominators, using objects or pictures.

Example: Use a picture of a circle divided into 6 equal pieces to find $\frac{5}{6} - \frac{1}{3}$.

4.2.9 Add and subtract decimals (to hundredths), using objects or pictures.

Example: Use coins to help you find \$0.43 − \$0.29.

4.2.10 Use a standard algorithm to add and subtract decimals (to hundredths).

Example: 0.74 + 0.80 = ?.

4.2.11 Know and use strategies for estimating results of any whole-number computations.

Example: Your friend says that 45,329 + 6,984 = 5,213. Without solving, explain why you think the answer is wrong.

4.2.12 Use mental arithmetic to add or subtract numbers rounded to hundreds or thousands.

Example: Add 3,000 to 8,000 without using pencil and paper.

* algorithm: a step-by-step procedure for solving a problem

Standard 3

Algebra and Functions

Students use and interpret variables, mathematical symbols, and properties to write and simplify numerical expressions and sentences. They understand relationships among the operations of addition, subtraction, multiplication, and division.

4.3.1 Use letters, boxes, or other symbols to represent any number in simple expressions, equations, or inequalities (i.e., demonstrate an understanding of and the use of the concept of a variable).

Example: You read the expression "three times some number added to five" and you write "$3x + 5$." What does x represent?

4.3.2 Use and interpret formulas to answer questions about quantities and their relationships.

Example: Write the formula for the area of a rectangle in words. Now let l stand for the length, w for the width, and A for the area. Write the formula using these symbols.

4.3.3 Understand that multiplication and division are performed before addition and subtraction in expressions without parentheses.

Example: You go to a store with 90¢ and buy 3 pencils that cost 20¢ each. Write an expression for the amount of money you have left and find its value.

4.3.4 Understand that an equation such as $y = 3x + 5$ is a rule for finding a second number when a first number is given.

Example: Use the formula $y = 3x + 5$ to find the value of y when $x = 6$.

4.3.5 Continue number patterns using multiplication and division.

Example: What is the next number: 160, 80, 40, 20, …? Explain your answer.

4.3.6 Recognize and apply the relationships between addition and multiplication, between subtraction and division, and the inverse relationship between multiplication and division to solve problems.

Example: Find another way of writing $13 + 13 + 13 + 13 + 13$.

4.3.7 Relate problem situations to number sentences involving multiplication and division.

Example: You have 150 jelly beans to share among the 30 members of your class. Write a number sentence for this problem and use it to find the number of jelly beans each person will get.

4.3.8 Plot and label whole numbers on a number line up to 100. Estimate positions on the number line.

Example: Draw a number line and label it with 0, 10, 20, 30, …, 90, 100. Estimate the position of 77 on this number line.

Standard 4

Geometry

Students show an understanding of plane and solid geometric objects and use this knowledge to show relationships and solve problems.

4.4.1 Identify, describe, and draw rays, right angles, acute angles, obtuse angles, and straight angles using appropriate mathematical tools and technology.

Example: Draw two rays that meet in an obtuse angle.

4.4.2 Identify, describe, and draw parallel, perpendicular, and oblique lines using appropriate mathematical tools and technology.

Example: Use the markings on the gymnasium floor to identify two lines that are parallel. Place a jump rope across the parallel lines and identify any obtuse angles created by the jump rope and the lines.

4.4.3 Identify, describe, and draw parallelograms*, rhombuses*, and trapezoids*, using appropriate mathematical tools and technology.

Example: Use a geoboard to make a parallelogram. How do you know it is a parallelogram?

4.4.4 Identify congruent* quadrilaterals* and give reasons for congruence using sides, angles, parallels, and perpendiculars.

Example: In a collection of parallelograms, rhombuses, and trapezoids, pick out those that are the same shape and size and explain your decisions.

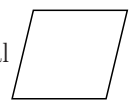
4.4.5 Identify and draw lines of symmetry in polygons.

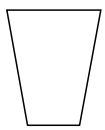
Example: Draw a rectangle and then draw all its lines of symmetry.

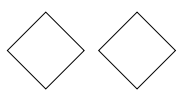
4.4.6 Construct cubes and prisms* and describe their attributes.

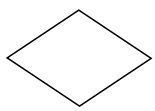
Example: Make a 6-sided prism from construction paper.

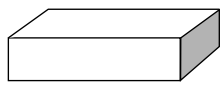
* parallelogram: a four-sided figure with both pairs of opposite sides parallel 

* rhombus: a parallelogram with all sides equal 

* trapezoid: a four-sided figure with one pair of opposite sides parallel 

* congruent: the term to describe two figures that are the same shape and size 

* quadrilateral: a two-dimensional figure with four sides 

* prism: a solid shape with fixed cross-section (a right prism is a solid shape with two parallel faces that are congruent polygons and other faces that are rectangles) 

Measurement

Students understand perimeter and area, as well as measuring volume, capacity, time, and money.

4.5.1 Measure length to the nearest quarter-inch, eighth-inch, and millimeter.

Example: Measure the width of a sheet of paper to the nearest millimeter.

4.5.2 Subtract units of length that may require renaming of feet to inches or meters to centimeters.

Example: The shelf was 2 feet long. Jane shortened it by 8 inches. How long is the shelf now?

4.5.3 Know and use formulas for finding the perimeters of rectangles and squares.

Example: The length of a rectangle is 4 cm and its perimeter is 20 cm. What is the width of the rectangle?

4.5.4 Know and use formulas for finding the areas of rectangles and squares.

Example: Draw a rectangle 5 inches by 3 inches. Divide it into one-inch squares and count the squares to find its area. Can you see another way to find the area? Do this with other rectangles.

4.5.5 Estimate and calculate the area of rectangular shapes using appropriate units, such as square centimeter (cm^2), square meter (m^2), square inch (in^2), or square yard (yd^2).

Example: Measure the length and width of a basketball court and find its area in suitable units.

4.5.6 Understand that rectangles with the same area can have different perimeters and that rectangles with the same perimeter can have different areas.

Example: Make a rectangle of area 12 units on a geoboard and find its perimeter. Can you make other rectangles with the same area? What are their perimeters?

4.5.7 Find areas of shapes by dividing them into basic shapes such as rectangles.

Example: Find the area of your school building.

4.5.8 Use volume and capacity as different ways of measuring the space inside a shape.

Example: Use cubes to find the volume of a fish tank and a pint jug to find its capacity.

4.5.9 Add time intervals involving hours and minutes.

Example: During the school week, you have 5 recess periods of 15 minutes. Find how long that is in hours and minutes.

4.5.10 Determine the amount of change from a purchase.

Example: You buy a chocolate bar priced at $1.75. How much change do you get if you pay for it with a five-dollar bill?

Standard 6

Data Analysis and Probability

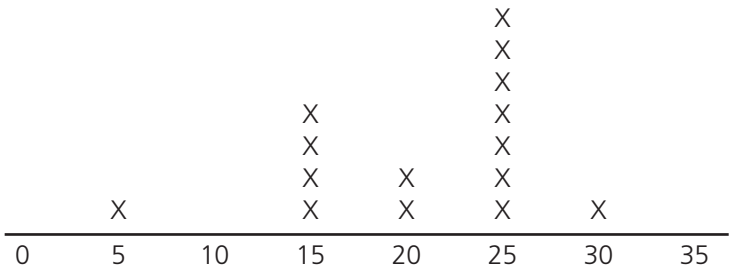
Students organize, represent, and interpret numerical and categorical data and clearly communicate their findings. They show outcomes for simple probability situations.

4.6.1 Represent data on a number line and in tables, including frequency tables.

Example: The students in your class are growing plants in various parts of the classroom. Plan a survey to measure the height of each plant in centimeters on a certain day. Record your survey results on a line plot.

4.6.2 Interpret data graphs to answer questions about a situation.

Example: The line plot below shows the heights of fast-growing plants reported by third-grade students. Describe any patterns that you can see in the data using the words "most," "few," and "none."



Plant Heights in Centimeters

4.6.3 Summarize and display the results of probability experiments in a clear and organized way.

Example: Roll a number cube 36 times and keep a tally of the number of times that 1, 2, 3, 4, 5, and 6 appear. Draw a bar graph to show your results.

Standard 7

Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

4.7.1 Analyze problems by identifying relationships, telling relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

Example: Solve the problem: "Find a relationship between the number of faces, edges, and vertices of a solid shape with flat surfaces." Try two or three shapes and look for patterns.

4.7.2 Decide when and how to break a problem into simpler parts.

Example: In the first example, find what happens to cubes and rectangular solids.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

4.7.3 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, use your method for cubes and rectangular solids to find what happens to other prisms and to pyramids.

4.7.4 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, tools, and models to solve problems, justify arguments, and make conjectures.

Example: In the first example, make a table to help you explain your results to another student.

4.7.5 Express solutions clearly and logically by using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, explain what happens with all the shapes that you tried.

4.7.6 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: You are telling a friend the time of a TV program. How accurate should you be: to the nearest day, hour, minute, or second?

4.7.7 Know and use appropriate methods for estimating results of whole-number computations.

Example: You buy 2 CDs for $15.95 each. The cashier tells you that will be $49.90. Does that surprise you?

4.7.8 Make precise calculations and check the validity of the results in the context of the problem.

Example: The buses you use for a school trip hold 55 people each. How many buses will you need to seat 180 people?

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

4.7.9 Decide whether a solution is reasonable in the context of the original situation.

Example: In the last example, would an answer of 3.27 surprise you?

4.7.10 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: Change the first example so that you look at shapes with curved surfaces.

NOTES

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 5:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students extend their understanding of the magnitudes of numbers to rounding whole numbers and decimals to any place value. They order and compare whole numbers and decimals using the correct symbols for greater than and less than. They develop the concept of percentage as parts of a hundred and compare different ways of looking at fractions. They identify whole numbers as prime or composite, and they compare fractions, decimals, and mixed numbers on a number line.

Standard 2 — Computation

Fluency in computation is essential. Students extend the standard methods for multiplying and dividing to larger numbers. They add and subtract more complex fractions and decimals, learning how these different representations of numbers can be manipulated. They also develop an understanding of how to multiply and divide fractions.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level develop further the fundamental concept of a variable — having a letter stand for all numbers of a certain kind. They use this to write simple algebraic expressions and to evaluate them. They begin to develop the idea of linking an algebraic equation to a graph, by finding ordered pairs that fit a linear equation, plotting these as points on a grid, and drawing the resulting straight line. They also interpret graphs to answer questions.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They draw angles, parallel and perpendicular lines, the radius and diameter of circles, and other geometric shapes, using ruler, compass, protractor, and computer drawing programs. They identify congruent triangles and explain their reasoning using specific geometrical terms, such as equilateral, isosceles, acute, and obtuse. They classify polygons with five or more sides. They develop an understanding of reflectional and rotational symmetry, and they construct prisms and pyramids, developing their ability to work in three dimensions.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students develop and use the formulas for calculating perimeters and areas of triangles, parallelograms, and trapezoids. They extend these ideas to finding the volume and surface area of rectangular solids. They understand and use additional units for measuring weight: ounce, gram, and ton. They also add and subtract with money in decimal notation.

Standard 6 — Data Analysis and Probability

Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they use the mean, median, mode, and range to describe data sets. They further develop the concept of probability, recording probabilities as fractions between 0 and 1 and linking these to levels of certainty about the events described.

Standard 7 — Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with algebra, geometry, or measurement, for example, students move from simple to more complex ideas by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense

Students compute with whole numbers, decimals, and fractions and understand the relationship among decimals, fractions, and percents. They understand the relative magnitudes of numbers. They understand prime* and composite* numbers.*

5.1.1 Convert between numbers in words and numbers in figures, for numbers up to millions and decimals to thousandths.

Example: Write the number 198.536 in words.

5.1.2 Round whole numbers and decimals to any place value.

Example: Is 7,683,559 closer to 7,600,000 or 7,700,000? Explain your answer.

5.1.3 Arrange in numerical order and compare whole numbers or decimals to two decimal places by using the symbols for less than (<), equals (=), and greater than (>).

Example: Write from smallest to largest: 0.5, 0.26, 0.08.

5.1.4 Interpret percents as a part of a hundred. Find decimal and percent equivalents for common fractions and explain why they represent the same value.

Example: Shade a 100-square grid to show 30%. What fraction is this?

5.1.5 Explain different interpretations of fractions: as parts of a whole, parts of a set, and division of whole numbers by whole numbers.

Example: What fraction of a pizza will each person get when 3 pizzas are divided equally among 5 people?

5.1.6 Describe and identify prime and composite numbers.

Example: Which of the following numbers are prime: 3, 7, 12, 17, 18? Justify your choices.

5.1.7 Identify on a number line the relative position of simple positive fractions, positive mixed numbers, and positive decimals.

Example: Find the positions on a number line of $1\frac{1}{4}$ and 1.4.

* whole number: 0, 1, 2, 3, etc.

* prime number: a number that can be evenly divided only by 1 and itself (e.g., 2, 3, 5, 7, 11)

* composite number: a number that is not a prime number (e.g., 4, 6, 8, 9, 10)

Standard 2

Computation

Students solve problems involving multiplication and division of whole numbers and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

5.2.1 Solve problems involving multiplication and division of any whole numbers.

Example: 2,867 × 34 = ?. Explain your method.

5.2.2 Add and subtract fractions (including mixed numbers) with different denominators.

Example: $3\frac{4}{5} - 2\frac{2}{3} = ?$.

5.2.3 Use models to show an understanding of multiplication and division of fractions.

Example: Draw a rectangle 5 squares wide and 3 squares high. Shade $\frac{4}{5}$ of the rectangle, starting from the left. Shade $\frac{2}{3}$ of the rectangle, starting from the top. Look at the fraction of the squares that you have double-shaded and use that to show how to multiply $\frac{4}{5}$ by $\frac{2}{3}$.

5.2.4 Multiply and divide fractions to solve problems.

Example: You have $3\frac{1}{2}$ pizzas left over from a party. How many people can have $\frac{1}{4}$ of a pizza each?

5.2.5 Add and subtract decimals and verify the reasonableness of the results.

Example: Compute 39.46 − 20.89 and check the answer by estimating.

5.2.6 Use estimation to decide whether answers are reasonable in addition, subtraction, multiplication, and division problems.

Example: Your friend says that 2,867 × 34 = 20,069. Without solving, explain why you think the answer is wrong.

5.2.7 Use mental arithmetic to add or subtract simple decimals.

Example: Add 0.006 to 0.027 without using pencil and paper.

Standard 3

Algebra and Functions

Students use variables in simple expressions, compute the value of an expression for specific values of the variable, and plot and interpret the results. They use two-dimensional coordinate grids to represent points and graph lines.

5.3.1 Use a variable to represent an unknown number.

Example: When a certain number is multiplied by 3 and then 5 is added, the result is 29. Let x stand for the unknown number and write an equation for the relationship.

5.3.2 Write simple algebraic expressions in one or two variables and evaluate them by substitution.

Example: Find the value of $5x + 2$ when $x = 3$.

5.3.3 Use the distributive property* in numerical equations and expressions.

Example: Explain how you know that $3(16 - 11) = 3 \times 16 - 3 \times 11$.

5.3.4 Identify and graph ordered pairs of positive numbers.

Example: Plot the points (3, 1), (6, 2), and (9, 3). What do you notice?

5.3.5 Find ordered pairs (positive numbers only) that fit a linear equation, graph the ordered pairs, and draw the line they determine.

Example: For $x = 1, 2, 3$, and 4, find points that fit the equation $y = 2x + 1$. Plot those points on graph paper and join them with a straight line.

5.3.6 Understand that the length of a horizontal line segment on a coordinate plane equals the difference between the x-coordinates and that the length of a vertical line segment on a coordinate plane equals the difference between the y-coordinates.

Example: Find the distance between the points (2, 5) and (7, 5) and the distance between the points (2, 1) and (2, 5).

5.3.7 Use information taken from a graph or equation to answer questions about a problem situation.

Example: The speed (v feet per second) of a car t seconds after it starts is given by the formula $v = 12t$. Find the car's speed after 5 seconds.

* distributive property: e.g., $3(5 + 2) = (3 \times 5) + (3 \times 2)$

Standard 4

Geometry

Students identify, describe, and classify the properties of plane and solid geometric shapes and the relationships between them.

5.4.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, triangles, and circles by using appropriate tools (e.g., ruler, compass, protractor, appropriate technology, media tools).

Example: Draw a rectangle with sides 5 inches and 3 inches.

5.4.2 Identify, describe, draw, and classify triangles as equilateral*, isosceles*, scalene*, right*, acute*, obtuse*, and equiangular*.

Example: Draw an isosceles right triangle.

5.4.3 Identify congruent* triangles and justify your decisions by referring to sides and angles.

Example: In a collection of triangles, pick out those that are the same shape and size and explain your decisions.

5.4.4 Identify, describe, draw, and classify polygons*, such as pentagons and hexagons.

Example: In a collection of polygons, pick out those with the same number of sides.

5.4.5 Identify and draw the radius and diameter of a circle and understand the relationship between the radius and diameter.

Example: On a circle, draw a radius and a diameter and describe the differences and similarities between the two.

5.4.6 Identify shapes that have reflectional and rotational symmetry*.

Example: What kinds of symmetries have the letters M, N, and O?

5.4.7 Understand that 90°, 180°, 270°, and 360° are associated with quarter, half, three-quarters, and full turns, respectively.

Example: Face the front of the room. Turn through four right angles. Which way are you now facing?

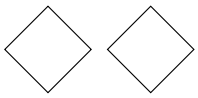
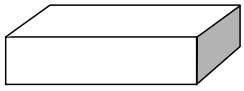
5.4.8 Construct prisms* and pyramids using appropriate materials.

Example: Make a square-based pyramid from construction paper.

5.4.9 Given a picture of a three-dimensional object, build the object with blocks.

Example: Given a picture of a house made of cubes and rectangular prisms, build the house.

* equilateral triangle: a triangle where all sides are congruent
* isosceles triangle: a triangle where at least two sides are congruent
* scalene triangle: a triangle where no sides are equal
* right triangle: a triangle where one angle measures 90 degrees
* acute triangle: a triangle where all angles are less than 90 degrees
* obtuse triangle: a triangle where one angle is more than 90 degrees
* equiangular triangle: a triangle where all angles are of equal measure

* congruent: the term to describe two figures that are the same shape and size 
* polygon: a two-dimensional shape with straight sides (e.g., triangle, rectangle, pentagon)
* reflectional and rotational symmetry: letter M has reflectional symmetry in a line down the middle; letter N has rotational symmetry around its center
* prism: a solid shape with fixed cross-section (a right prism is a solid shape with two parallel faces that are congruent polygons and other faces that are rectangles) 

Standard 5

Measurement

Students understand and compute the areas and volumes of simple objects, as well as measuring weight, temperature, time, and money.

5.5.1 Understand and apply the formulas for the area of a triangle, parallelogram, and trapezoid.

Example: Find the area of a triangle with base 4 m and height 5 m.

5.5.2 Solve problems involving perimeters and areas of rectangles, triangles, parallelograms, and trapezoids, using appropriate units.

Example: A trapezoidal garden bed has parallel sides of lengths 14 m and 11 m and its width is 6 m. Find its area and the length of fencing needed to enclose it. Be sure to use correct units.

5.5.3 Use formulas for the areas of rectangles and triangles to find the area of complex shapes by dividing them into basic shapes.

Example: A square room of length 17 feet has a tiled fireplace area that is 6 feet long and 4 feet wide. You want to carpet the floor of the room, except the fireplace area. Find the area to be carpeted.

5.5.4 Find the surface area and volume of rectangular solids using appropriate units.

Example: Find the volume of a shoe box with length 30 cm, width 15 cm, and height 10 cm.

5.5.5 Understand and use the smaller and larger units for measuring weight (ounce, gram, and ton) and their relationship to pounds and kilograms.

Example: How many ounces are in a pound?

5.5.6 Compare temperatures in Celsius and Fahrenheit, knowing that the freezing point of water is 0°C and 32°F and that the boiling point is 100°C and 212°F.

Example: What is the Fahrenheit equivalent of 50°C? Explain your answer.

5.5.7 Add and subtract with money in decimal notation.

Example: You buy articles that cost $3.45, $6.99, and $7.95. How much change will you receive from $20?

Standard 6

Data Analysis and Probability

Students collect, display, analyze, compare, and interpret data sets. They use the results of probability experiments to predict future events.

5.6.1 Explain which types of displays are appropriate for various sets of data.

Example: Conduct a survey to find the favorite movies of the students in your class. Decide whether to use a bar, line, or picture graph to display the data. Explain your decision.

5.6.2 Find the mean*, median*, mode*, and range* of a set of data and describe what each does and does not tell about the data set.

Example: Find the mean, median, and mode of a set of test results and describe how well each represents the data.

5

5.6.3 Understand that probability can take any value between 0 and 1, events that are not going to occur have probability 0, events certain to occur have probability 1, and more likely events have a higher probability than less likely events.

Example: What is the probability of rolling a 7 with a number cube?

5.6.4 Express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4, $\frac{3}{4}$).

Example: What is the probability of rolling an odd number with a number cube?

* mean: the average obtained by adding the values and dividing by the number of values
* median: the value that divides a set of data, written in order of size, into two equal parts
* mode: the most common value in a given data set
* range: the difference between the largest and smallest values

Standard 7

Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

5.7.1 Analyze problems by identifying relationships, telling relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

Example: Solve the problem: "When you flip a coin 3 times, you can get 3 heads, 3 tails, 2 heads and 1 tail, or 1 head and 2 tails. Find the probability of each of these combinations." Notice that the case of 3 heads and the case of 3 tails are similar. Notice that the case of 2 heads and 1 tail and the case of 1 head and 2 tails are similar.

5.7.2 Decide when and how to break a problem into simpler parts.

Example: In the first example, decide to look at the case of 3 heads and the case of 2 heads and 1 tail.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

5.7.3 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, begin with the situation where you flip the coin twice.

5.7.4 Express solutions clearly and logically by using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, make a table or tree diagram to show another student what is happening.

5.7.5 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: You are buying a piece of plastic to cover the floor of your bedroom before you paint the room. How accurate should you be: to the nearest inch, foot, or yard? Explain your answer.

5.7.6 Know and apply appropriate methods for estimating results of rational-number computations.

Example: Will 7×18 be smaller or larger than 100? Explain your answer.

5.7.7 Make precise calculations and check the validity of the results in the context of the problem.

Example: A recipe calls for $\frac{3}{8}$ of a cup of sugar. You plan to double the recipe for a party and you have only one cup of sugar in the house. Decide whether you have enough sugar and explain how you know.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

5.7.8 Decide whether a solution is reasonable in the context of the original situation.

Example: In the first example about flipping a coin, check that your probabilities add to 1.

5.7.9 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: Find the probability of each of the combinations when you flip a coin 4 times.

NOTES

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 6:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students continue to develop their understanding of the relationship between fractions and decimals. They extend the number system to include negative numbers. They also relate percentages to fractions and decimals and begin learning how to use ratios. They find multiples and factors of whole numbers, using the multiples and factors to solve problems involving fractions.

Standard 2 — Computation

Fluency in computation is essential. Students add, subtract, multiply, and divide fractions, decimals, and both positive and negative integers. They solve problems using ratios, proportions, and percentages, including calculating discount and interest. They use mental arithmetic to add or subtract simple fractions and decimals.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level write and solve simple equations and inequalities, and write and use formulas to solve problems. They use parentheses in more complex expressions to show the order of operations. They also extend graphs of straight lines to include negative values.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They draw special types of angles and use them to solve problems. They find and use the sum of the angles of a triangle and of a quadrilateral. They identify shapes that are similar (the same shape but not necessarily the same size). They draw reflections and translations of shapes, and they also draw two-dimensional views of three-dimensional shapes.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students measure in order to compare lengths, areas, volumes, weights, times, temperatures, etc. They learn about the number π and use it to calculate the circumference and area of circles. They construct models, find the volume and surface area of prisms and cylinders, and they convert temperatures between Celsius and Fahrenheit.

Standard 6 — Data Analysis and Probability

Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they learn how to display data in frequency tables and in stem-and-leaf plots. They compare the mean, median, and mode. They find probabilities for compound events and write them as fractions, decimals, and percentages. They also estimate the probabilities of future events.

Standard 7 — Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with negative numbers, calculating angles, or finding areas, for example, students move from simple to more complex ideas by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense

Students compare and order positive and negative integers, decimals, fractions, and mixed numbers. They find multiples* and factors*.*

6.1.1 Understand and apply the basic concept of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").

Example: The temperature this morning was -6° and now it is 3°. How much has the temperature risen? Explain your answer.

6.1.2 Interpret the absolute value of a number as the distance from zero on a number line and find the absolute value of real numbers.

Example: Use a number line to explain the absolute values of -3 and of 7.

6.1.3 Compare and represent on a number line positive and negative integers, fractions, decimals (to hundredths), and mixed numbers.

Example: Find the positions on a number line of 3.56, -2.5, $1\frac{5}{6}$, and -4.

6.1.4 Convert between any two representations of numbers (fractions, decimals, and percents) without the use of a calculator.

Example: Write $\frac{5}{8}$ as a decimal and as a percent.

6.1.5 Recognize decimal equivalents for commonly used fractions without the use of a calculator.

Example: Know that $\frac{1}{3} = 0.333\ldots$, $\frac{1}{2} = 0.5$, $\frac{2}{5} = 0.4$, etc.

6.1.6 Use models to represent ratios.

Example: Divide 27 pencils to represent the ratio 4:5.

6.1.7 Find the least common multiple* and the greatest common factor* of whole numbers. Use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Example: Find the smallest number that both 12 and 18 divide into. How does this help you add the fractions $\frac{5}{12}$ and $\frac{7}{18}$?

* positive and negative integers: …, -3, -2, -1, 0, 1, 2, 3, …

* multiples: e.g., multiples of 7 are 7, 14, 21, 28, etc.

* factors: e.g., factors of 12 are 1, 2, 3, 4, 6, 12

* least common multiple: e.g., the least common multiple of 4 and 6 is 12

* greatest common factor: e.g., the greatest common factor of 18 and 42 is 6

Standard 2

Computation

Students solve problems involving addition, subtraction, multiplication, and division of integers. They solve problems involving fractions, decimals, ratios, proportions, and percentages.

6.2.1 Add and subtract positive and negative integers.

Example: $17 + -4 = ?$, $-8 - 5 = ?$.

6.2.2 Multiply and divide positive and negative integers.

Example: Continue the pattern: $3 \times 2 = ?$, $2 \times 2 = ?$, $1 \times 2 = ?$, $0 \times 2 = ?$, $-1 \times 2 = ?$, $-2 \times 2 = ?$, etc.

6.2.3 Multiply and divide decimals.

Example: $3.265 \times 0.96 = ?$, $56.79 \div 2.4 = ?$.

6.2.4 Explain how to multiply and divide positive fractions and perform the calculations.

Example: Explain why $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15} = \frac{2}{3}$.

6.2.5 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

Example: You want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door $27\frac{1}{2}$ inches wide. How far from each edge should you place the bar? Explain your method.

6.2.6 Interpret and use ratios to show the relative sizes of two quantities. Use the notations: a/*b*, *a* to *b*, *a*:*b*.

Example: A car moving at a constant speed travels 130 miles in 2 hours. Write the ratio of distance to time and use it to find how far the car will travel in 5 hours.

6.2.7 Understand proportions and use them to solve problems.

Example: Sam made 8 out of 24 free throws. Use a proportion to show how many free throws Sam would probably make out of 60 attempts.

6.2.8 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

Example: In a sale, everything is reduced by 20%. Find the sale price of a shirt whose pre-sale price was $30.

6.2.9 Use estimation to decide whether answers are reasonable in decimal problems.

Example: Your friend says that $56.79 \div 2.4 = 2.36625$. Without solving, explain why you think the answer is wrong.

6.2.10 Use mental arithmetic to add or subtract simple fractions and decimals.

Example: Subtract $\frac{1}{6}$ from $\frac{1}{2}$ without using pencil and paper.

Standard 3

Algebra and Functions

Students write verbal expressions and sentences as algebraic expressions and equations. They evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results. They investigate geometric relationships and describe them algebraically.

6.3.1 Write and solve one-step linear equations and inequalities in one variable and check the answers.

Example: The area of a rectangle is 143 cm^2 and the length is 11 cm. Write an equation to find the width of the rectangle and use it to solve the problem. Describe how you will check to be sure that your answer is correct.

6.3.2 Write and use formulas with up to three variables to solve problems.

Example: You have P dollars in a bank that gives $r\%$ simple interest per year. Write a formula for the amount of interest you will receive in one year. Use the formula to find the amount of interest on \$80 at 6% per year for one year.

6.3.3 Interpret and evaluate expressions that use grouping symbols such as parentheses.

Example: Find the values of $10 - (7 - 3)$ and of $2(10 - 7)(3 + 1)$.

6.3.4 Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.

Example: Write in symbols: add 19 and 34 and double the result.

6.3.5 Use variables in expressions describing geometric quantities.

Example: Let l, w, and P be the length, width, and perimeter of a rectangle. Write a formula for the perimeter in terms of the length and width.

6.3.6 Apply the correct order of operations and the properties of real numbers (e.g., identity, inverse, commutative*, associative*, and distributive* properties) to evaluate numerical expressions. Justify each step in the process.

Example: Simplify $3(4 - 1) + 2$. Explain your method.

6.3.7 Identify and graph ordered pairs in the four quadrants of the coordinate plane.

Example: Plot the points (3, -1), (-6, 2) and (9, -3). What do you notice?

6.3.8 Solve problems involving linear functions with integer* values. Write the equation and graph the resulting ordered pairs of integers on a grid.

Example: A plant is 3 cm high the first time you measure it (on Day 0). Each day after that the plant grows by 2 cm. Write an equation connecting the height and the number of the day and draw its graph.

6.3.9 Investigate how a change in one variable relates to a change in a second variable.

Example: In the last example, what do you notice about the shape of the graph?

* commutative property: the order when adding or multiplying numbers makes no difference (e.g., $5 + 3 = 3 + 5$), but note that this is not true for subtraction or division

* associative property: the grouping when adding or multiplying numbers makes no difference (e.g., in $5 + 3 + 2$, adding 5 and 3 and then adding 2 is the same as 5 added to $3 + 2$), but note that this is not true for subtraction or division

* distributive property: e.g., $3(5 + 2) = (3 \times 5) + (3 \times 2)$

* integer: …, -3, -2, -1, 0, 1, 2, 3, …

Standard 4

Geometry

Students identify, describe, and classify the properties of plane and solid geometric shapes and the relationships between them.

6.4.1 Identify and draw vertical*, adjacent*, complementary*, and supplementary* angles and describe these angle relationships.

Example: Draw two parallel lines with another line across them. Identify all pairs of supplementary angles.

6.4.2 Use the properties of complementary, supplementary, and vertical angles to solve problems involving an unknown angle. Justify solutions.

Example: Find the size of the supplement to an angle that measures 122º. Explain how you obtain your answer.

6.4.3 Draw quadrilaterals* and triangles from given information about them.

Example: Draw a quadrilateral with equal sides but no right angles.

6.4.4 Understand that the sum of the interior angles of any triangle is 180º and that the sum of the interior angles of any quadrilateral is 360º. Use this information to solve problems.

Example: Find the size of the third angle of a triangle with angles of 73º and 49º.

6.4.5 Identify and draw two-dimensional shapes that are similar*.

Example: Draw a rectangle similar to a given rectangle, but twice the size.

6.4.6 Draw the translation (slide) and reflection (flip) of shapes.

Example: Draw a square and then slide it 3 inches horizontally across your page. Draw the new square in a different color.

6.4.7 Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Example: Draw a picture of an arrangement of rectangular blocks from the top, front, and right-hand side.

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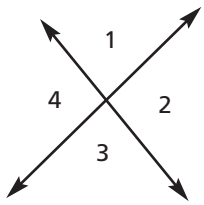
* vertical angles: angles 1 and 3 or 2 and 4

* adjacent angles: angles 1 and 2 or 2 and 3, etc.

* complementary angles: two angles whose sum is 90º

* supplementary angles: two angles whose sum is 180º (angles 1 and 2)



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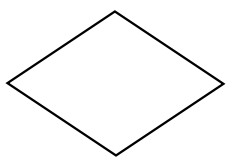
* quadrilateral: a two-dimensional figure with four sides



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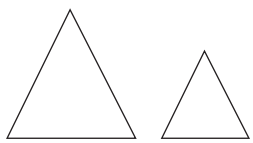
* similar: the term to describe figures that have the same shape but may not have the same size



Standard 5

Measurement

Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems. They calculate with temperature and money, and choose appropriate units of measure in other areas.

6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.

Example: A triangular sheet of metal is about 1 foot across. Describe the units and tools you would use to measure its weight, its angles, and the lengths of its sides.

6.5.2 Understand and use larger units for measuring length by comparing miles to yards and kilometers to meters.

Example: How many meters are in a kilometer?

6.5.3 Understand and use larger units for measuring area by comparing acres and square miles to square yards and square kilometers to square meters.

Example: How many square meters are in a square kilometer?

6.5.4 Understand the concept of the constant π as the ratio of the circumference to the diameter of a circle. Develop and use the formulas for the circumference and area of a circle.

Example: Measure the diameter and circumference of several circular objects. (Use string to find the circumference.) With a calculator, divide each circumference by its diameter. What do you notice about the results?

6.5.5 Know common estimates of π (3.14, $^{22}/_{7}$) and use these values to estimate and calculate the circumference and the area of circles. Compare with actual measurements.

Example: Find the area of a circle of radius 15 cm.

6.5.6 Understand the concept of significant figures and round answers to an appropriate number of significant figures.

Example: You measure the diameter of a circle as 2.47 m and use the approximation 3.14 for π to calculate the circumference. Is it reasonable to give 7.7558 m as your answer? Why or why not?

6.5.7 Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area of these objects.

Example: Find the total surface area of a shoe box with length 30 cm, width 15 cm, and height 10 cm.

6.5.8 Use strategies to find the surface area and volume of right prisms* and cylinders using appropriate units.

Example: Find the volume of a cylindrical can 15 cm high and with a diameter of 8 cm.

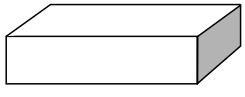
6.5.9 Use a formula to convert temperatures between Celsius and Fahrenheit.

Example: What is the Celsius equivalent of 100°F? Explain your method.

6.5.10 Add, subtract, multiply, and divide with money in decimal notation.

Example: Share $7.25 among five people.

* right prism: a three-dimensional shape with two congruent ends that are polygons and all other faces are rectangles



Standard 6

Data Analysis and Probability

Students compute and analyze statistical measures for data sets. They determine theoretical and experimental probabilities and use them to make predictions about events.

6.6.1 Organize and display single-variable data in appropriate graphs and stem-and-leaf plots*, and explain which types of graphs are appropriate for various data sets.

Example: This stem-and-leaf diagram shows a set of test scores for your class:

Stem	Leaf
6	2 3 7
7	1 5 5 6 8 9
8	0 1 1 2 3 3 5 7 8 8
9	1 2 2 3 3 4

Find your score of 85 in this diagram. Are you closer to the top or the bottom of the class on this test?

6

6.6.2 Make frequency tables for numerical data, grouping the data in different ways to investigate how different groupings describe the data. Understand and find relative and cumulative frequency for a data set. Use histograms of the data and of the relative frequency distribution, and a broken line graph for cumulative frequency, to interpret the data.

Example: A bag contains pens in three colors. Nine students each draw a pen from the bag without looking, then record the results in the frequency table shown. Complete the column showing relative frequency.

Color	Frequency	Relative Frequency
Red	2	2/9
Green	4	
Purple	3	

6.6.3 Compare the mean*, median*, and mode* for a set of data and explain which measure is most appropriate in a given context.

Example: Twenty students were given a science test and the mean, median and mode were as follows:

mean = 8.5, median = 9, mode = 10.

What does the difference between the mean and the mode suggest about the twenty quiz scores?

6.6.4 Show all possible outcomes for compound events in an organized way and find the theoretical probability of each outcome.

Example: A box contains four cards with the numbers 1 through 4 written on them. Show a list of all the possible outcomes if you draw two cards from the box without looking. What is the theoretical probability that you will draw the numbers one and two? Explain your answer.

6.6.5 Use data to estimate the probability of future events.

Example: Teams A and B have played each other 3 times this season and Team A has won twice. When they play again, what is the probability of Team B winning? How accurate do you think this estimate is?

6.6.6 Understand and represent probabilities as ratios, measures of relative frequency, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable.

Example: The weather forecast says that the chance of rain today is 30%. Should you carry an umbrella? Explain your answer.

* stem-and-leaf plot: the example under 6.6.1 shows 62, 63, 67, 71, 75, 75, 76, etc.
* mean: the average obtained by adding the values and dividing by the number of values
* median: the value that divides a set of data, written in order of size, into two equal parts
* mode: the most common value in a given data set

Standard 7

Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

6.7.1 Analyze problems by identifying relationships, telling relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

Example: Solve the problem: "Develop a method for finding all the prime numbers up to 100." Notice that any numbers that 4, 6, 8, … divide into also divide exactly by 2, and so you do not need to test 4, 6, 8, … .

6.7.2 Make and justify mathematical conjectures based on a general description of a mathematical question or problem.

Example: In the first example, decide that you need to test only the prime numbers as divisors, and explain it in the same way as for 4, 6, 8, … .

6.7.3 Decide when and how to break a problem into simpler parts.

Example: In the first example, decide to find first those numbers not divisible by 2.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

6.7.4 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, begin by finding all the prime numbers up to 10.

6.7.5 Express solutions clearly and logically by using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, use a hundreds chart to cross off all multiples of 2 (except 2), then all multiples of 3 (except 3), then all multiples of 5 (except 5), etc. Explain why you are doing this.

6.7.6 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: Calculate the perimeter of a rectangular field that needs to be fenced. How accurate should you be: to the nearest kilometer, meter, centimeter, or millimeter? Explain your answer.

6.7.7 Select and apply appropriate methods for estimating results of rational-number computations.

Example: Measure the length and height of the walls of a room to find the total area. Estimate an answer by imagining meter squares covering the walls.

6.7.8 Use graphing to estimate solutions and check the estimates with analytic approaches.

Example: Use a graphing calculator to estimate the coordinates of the point where the straight line $y = 8x - 3$ crosses the x-axis. Confirm your answer by checking it in the equation.

6.7.9 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, check whether some of the numbers not crossed out are in fact primes.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

6.7.10 Decide whether a solution is reasonable in the context of the original situation.

Example: In the first example, decide whether your method was a good one — did it find all the prime numbers efficiently?

6.7.11 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: Use a hundreds chart to find all the numbers that are multiples of both 2 and 3.

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 7:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students extend this understanding to include irrational numbers, such as π and the square root of 2. They compare and order rational and irrational numbers and convert terminating decimals into fractions. They also use exponents to write whole numbers in scientific notation and to write the prime factorizations of numbers.

Standard 2 — Computation

Fluency in computation is essential. Students add, subtract, multiply, and divide integers, fractions, and decimals. They solve problems using percentages, including calculating discounts, markups, and commissions. They use mental arithmetic to compute with simple fractions, decimals, and powers.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level use variables and other symbols to translate verbal descriptions into equations and formulas. They write and solve linear equations and inequalities, and write and use formulas to solve problems. They also use properties of the rational numbers to evaluate and simplify algebraic expressions, and they further extend their understanding of graphs by investigating rates of change for linear and nonlinear functions and by developing and using the concept of the slope of a straight line.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They link geometry to coordinate graphs, using them to plot shapes, calculate lengths and areas, and find images under transformations. They understand the Pythagorean Theorem and use it to find lengths in right triangles. They also construct nets (two-dimensional patterns) for three-dimensional objects, such as prisms, pyramids, cylinders, and cones.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students measure in order to compare lengths, areas, volumes, weights, times, temperatures, etc. They develop the concept of similarity and use it to make scale drawings and scale models and to solve problems relating to these drawings and models. They find areas and perimeters of two-dimensional shapes and volumes and surface areas of three-dimensional shapes, including irregular shapes made up of more basic shapes.

Standard 6 — Data Analysis and Probability

Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they learn how to display data in bar, line, and circle graphs and in stem-and-leaf plots. They analyze data displays to find whether they are misleading and analyze the wording of survey questions to tell whether these could influence the results. They find the probability of disjoint events. They also find the number of arrangements of objects using a tree diagram.

Standard 7 — Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with irrational numbers, analyzing graphs, or finding surface areas, for example, students move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

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Standard 1

Number Sense

Students understand and use scientific notation and square roots. They convert between fractions and decimals.*

7.1.1 Read, write, compare, and solve problems using whole numbers in scientific notation.

Example: Write 300,000 in scientific notation.

7.1.2 Compare and order rational* and common irrational* numbers and place them on a number line.

Example: Place in order: -2, $\frac{5}{8}$, -2.45, 0.9, π, $-1\frac{3}{4}$.

7.1.3 Identify rational and common irrational numbers from a list.

Example: Name all the irrational numbers in the list: -2, $\frac{5}{8}$, -2.45, 0.9, π, $-1\frac{3}{4}$.

7.1.4 Understand and compute whole number powers of whole numbers.

Example: $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = ?$.

7.1.5 Find the prime factorization* of whole numbers and write the results using exponents.

Example: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$.

7.1.6 Understand and apply the concept of square root.

Example: Explain how you can find the length of the hypotenuse of a right triangle with legs that measure 5 cm and 12 cm.

7.1.7 Convert terminating decimals* into reduced fractions.

Example: Write 0.95 as a fraction.

* scientific notation: a shorthand way of writing numbers using powers of ten (e.g., $300{,}000 = 3 \times 10^5$)

* rational number: a real number that can be written as a ratio of two integers* (e.g., $\frac{1}{2}$, $\frac{5}{6}$, $\frac{23}{9}$)

* integer: …, -3, -2, -1, 0, 1, 2, 3, …

* irrational number: a real number that cannot be written as a ratio of two integers (e.g., π, $\sqrt{3}$, 7π)

* prime factors: e.g., prime factors of 12 are 2 and 3, the two prime numbers that divide 12

* terminating decimals: decimals that do not continue indefinitely (e.g., 0.362, 34.1857)

Standard 2

Computation

Students solve problems involving integers, fractions, decimals, ratios, and percentages.*

7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.

Example: The temperature one day is 5°. It then falls by 3° each day for 4 days and, after that, rises by 2° each day for 3 days. What is the temperature on the last day? Explain your method.

7.2.2 Calculate the percentage increase and decrease of a quantity.

Example: The population of a country was 36 million in 1990 and it rose to 41.4 million during the 1990s. What was the percentage increase in the population?

7.2.3 Solve problems that involve discounts, markups, and commissions.

Example: A merchant buys CDs for $11 wholesale and marks up the price by 35%. What is the retail price?

7.2.4 Use estimation to decide whether answers are reasonable in problems involving fractions and decimals.

Example: Your friend says that $3\frac{3}{8} \times 2\frac{2}{9} = 10$. Without solving, explain why you think the answer is wrong.

7.2.5 Use mental arithmetic to compute with simple fractions, decimals, and powers.

Example: Find 3^4 without using pencil and paper.

* integer: …, -3, -2, -1, 0, 1, 2, 3, …

Standard 3

Algebra and Functions

Students express quantitative relationships using algebraic terminology, expressions, equations, inequalities, and graphs.

7.3.1 Use variables and appropriate operations to write an expression, a formula, an equation, or an inequality that represents a verbal description.

Example: Write in symbols the inequality: 5 less than twice the number is greater than 42.

7.3.2 Write and solve two-step linear equations and inequalities in one variable and check the answers.

Example: Solve the equation $4x - 7 = 12$ and check your answer in the original equation.

7.3.3 Use correct algebraic terminology, such as variable, equation, term, coefficient*, inequality, expression, and constant.

Example: Name the variable, terms, and coefficient in this equation: $7x + 4 = 67$.

7.3.4 **Evaluate numerical expressions and simplify algebraic expressions by applying the correct order of operations and the properties of rational numbers* (e.g., identity, inverse, commutative*, associative*, distributive properties*). Justify each step in the process.**

Example: Simplify $3(4x + 5x - 1) + 2(x + 3)$ by removing the parentheses and rearranging. Explain each step you take.

7.3.5 **Solve an equation or formula with two variables for a particular variable.**

Example: Solve the formula $C = 2\pi r$ for r.

7.3.6 **Define slope as vertical change per unit of horizontal change and recognize that a straight line has constant slope or rate of change.**

Example: Examine a table of values and make a conjecture about whether the table represents a linear function.

7.3.7 **Find the slope of a line from its graph.**

Example: Draw the graph of $y = 2x - 1$. Choose two points on the graph and divide the change in y-value by the change in x-value. Repeat this for other pairs of points on the graph. What do you notice?

7.3.8 **Draw the graph of a line given the slope and one point on the line, or two points on the line.**

Example: Draw the graph of the equation with slope of 3 and passing through the point with coordinates (0, -2).

7.3.9 **Identify functions as linear or nonlinear and examine their characteristics in tables, graphs, and equations.**

Example: A plant is growing taller according to the formula $H = 2d + 3$, where H is the height after d days. Draw the graph of this function and explain what the point where it meets the vertical axis represents. Is this graph linear or nonlinear?

7.3.10 **Identify and describe situations with constant or varying rates of change and know that a constant rate of change describes a linear function.**

Example: In the last example, how will the graph be different if the plant's speed of growth changes?

* coefficient: e.g., 7 is the coefficient in $7x$

* rational number: a real number that can be written as a ratio of two integers* (e.g., $\frac{1}{2}$, $\frac{5}{6}$, $\frac{23}{9}$)

* integer: …, -3, -2, -1, 0, 1, 2, 3, …

* commutative property: the order when adding or multiplying numbers makes no difference (e.g., $5 + 3 = 3 + 5$), but note that this is not true for subtraction or division

* associative property: the grouping when adding or multiplying numbers makes no difference (e.g., in $5 + 3 + 2$, adding 5 and 3 and then adding 2 is the same as 5 added to $3 + 2$), but note that this is not true for subtraction or division

* distributive property: e.g., $3(5 + 2) = (3 \times 5) + (3 \times 2)$

Standard 4

Geometry

Students deepen their understanding of plane and solid geometric shapes by constructing shapes that meet given conditions and by identifying attributes of shapes.

7.4.1 Understand coordinate graphs and use them to plot simple shapes, find lengths and areas related to the shapes, and find images under translations (slides), rotations (turns), and reflections (flips).

Example: Draw the triangle with vertices (0, 0), (3, 0), and (0, 4). Find the lengths of the sides and the area of the triangle. Translate (slide) the triangle 2 units to the right. What are the coordinates of the new triangle?

7.4.2 Understand that transformations such as slides, turns, and flips preserve the length of segments, and that figures resulting from slides, turns, and flips are congruent* to the original figures.

Example: In the last example, find the lengths of the sides and the area of the new triangle. Discuss your results.

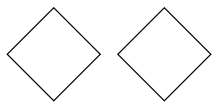
7.4.3 Know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and the lengths of other line segments. Use direct measurement to test conjectures about triangles.

Example: Use the length and width of your classroom to calculate the distance across the room diagonally. Check by measuring.

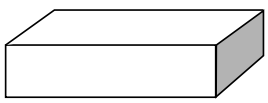
7.4.4 Construct two-dimensional patterns (nets) for three-dimensional objects, such as right prisms*, pyramids, cylinders, and cones.

Example: Draw a rectangle and two circles that will fit together to make a cylinder.

* congruent: the term to describe two figures that are the same shape and size



* right prism: a three-dimensional shape with two congruent ends that are polygons and all other faces are rectangles



Standard 5

Measurement

Students compare units of measure and use similarity to solve problems. They compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less regular objects.*

7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.

Example: The area of the school field is 3 acres. How many square yards is that? Explain your method.

7.5.2 Use experimentation and modeling to visualize similarity problems. Solve problems using similarity.

Example: At a certain time, the shadow of your school building is 36 feet long. At the same time, the shadow of a yardstick held vertically is 4 feet long. How high is the school building?

7.5.3 **Read and create drawings made to scale, construct scale models, and solve problems related to scale.**

Example: On a plan of your school, your classroom is 5 cm long and 3 cm wide. The actual classroom is 10 m long. How wide is it? Explain your answer.

7.5.4 **Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms*, trapezoids*, triangles, circles, right prisms*, and cylinders.**

Example: Find the surface area of a cylindrical can 15 cm high and with a diameter of 8 cm.

7.5.5 **Estimate and compute the area of more complex or irregular two-dimensional shapes by dividing them into more basic shapes.**

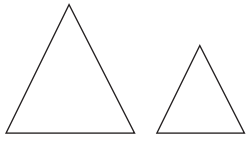
Example: A room to be carpeted is a rectangle 5 m × 4 m. A semicircular fireplace of diameter 1.5 m takes up some of the floor space. Find the area to be carpeted.

7.5.6 **Use objects and geometry modeling tools to compute the surface area of the faces and the volume of a three-dimensional object built from rectangular solids.**

Example: Build a model of an apartment building with blocks. Find its volume and total surface area.

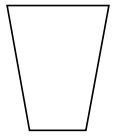
7

* similar: the term to describe figures that have the same shape but may not have the same size

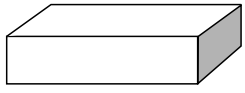


* parallelogram: a four-sided figure with both pairs of opposite sides parallel

* trapezoid: a four-sided figure with one pair of opposite sides parallel



* right prism: a three-dimensional shape with two congruent ends that are polygons and all other faces are rectangles



Standard 6

Data Analysis and Probability

Students collect, organize, and represent data sets and identify relationships among variables within a data set. They determine probabilities and use them to make predictions about events.

7.6.1 Analyze, interpret, and display data in appropriate bar, line, and circle graphs and stem-and-leaf plots* and justify the choice of display.

Example: You survey the students in your school to find which of three designs for a magazine cover they prefer. To display the results, which would be more appropriate: a bar chart or a circle graph? Explain your answer.

7.6.2 Make predictions from statistical data.

Example: Record the temperature and weather conditions (sunny, cloudy, or rainy) at 1 p.m. each day for two weeks. In the third week, use your results to predict the temperature from the weather conditions.

7.6.3 Describe how additional data, particularly outliers, added to a data set may affect the mean*, median*, and mode*.

Example: You measure the heights of the students in your grade on a day when the basketball team is playing an away game. Later you measure the players on the team and include them in your data. What kind of effect will including the team have on the mean, median, and mode? Explain your answer.

7

7.6.4 Analyze data displays, including ways that they can be misleading. Analyze ways in which the wording of questions can influence survey results.

Example: On a bar graph of a company's sales, it appears that sales have more than doubled since last year. Then you notice that the vertical axis starts at $5 million and can see that sales have in fact increased from $5.5 million to $6.2 million.

7.6.5 Know that if P is the probability of an event occurring, then $1 - P$ is the probability of that event not occurring.

Example: The weather forecast says that the probability of rain today is 0.3. What is the probability that it won't rain?

7.6.6 Understand that the probability of either one or the other of two disjoint events* occurring is the sum of the two individual probabilities.

Example: Find the probability of rolling 9 with two number cubes. Also find the probability of rolling 10. What is the probability of rolling 9 or 10?

7.6.7 Find the number of possible arrangements of several objects using a tree diagram.

Example: A state's license plates contain 6 digits and one letter. How many different license plates can be made if the letter must always be in the third position and the first digit cannot be a zero?

* stem-and-leaf plot: e.g., this one shows 62, 63, 67, 71, 75, 75, 76, etc.

Stem	Leaf
6	2 3 7
7	1 5 5 6 8 9
8	0 1 1 2 3 5 5 7 8 8
9	1 2 2 3 3 4

* mean: the average obtained by adding the values and dividing by the number of values
* median: the value that divides a set of data, written in order of size, into two equal parts
* mode: the most common value in a given data set
* disjoint events: events that cannot happen at the same time

Standard 7

Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

7.7.1 **Analyze problems by identifying relationships, telling relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.**

Example: Solve the problem: "The first three triangular numbers are shown in the diagram below. Find an expression to calculate the nth triangular number."

1 3 6

Decide to look for patterns.

7.7.2 **Make and justify mathematical conjectures based on a general description of a mathematical question or problem.**

Example: In the first example, notice that three dots make an equilateral triangle for the number 3 and six dots make the next equilateral triangle.

7.7.3 **Decide when and how to divide a problem into simpler parts.**

Example: In the first example, decide to make a diagram for the fourth and fifth triangular numbers.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

7.7.4 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, list the differences between any two triangular numbers.

7.7.5 Make and test conjectures by using inductive reasoning.

Example: In the first example, predict the difference between the fifth and sixth numbers and use this to predict the sixth triangular number. Make a diagram to test your conjecture.

7.7.6 Express solutions clearly and logically by using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, use words, numbers, and tables to summarize your work with triangular numbers.

7.7.7 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: Calculate the amount of aluminum needed to make a can with diameter 10 cm that is 15 cm high and 1 mm thick. Take π as 3.14 and give your answer to appropriate accuracy.

7

7.7.8 Select and apply appropriate methods for estimating results of rational-number computations.

Example: Measure the dimensions of a swimming pool to find its volume. Estimate an answer by working with an average depth.

7.7.9 Use graphing to estimate solutions and check the estimates with analytic approaches.

Example: Use a graphing calculator to find the crossing point of the straight lines $y = 2x + 3$ and $x + y = 10$. Confirm your answer by checking it in the equations.

7.7.10 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, check that your later results fit with your earlier ones. If they do not, repeat the calculations to make sure.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

7.7.11 Decide whether a solution is reasonable in the context of the original situation.

Example: In the first example, calculate the 10^{th} triangular number and draw the triangle of dots that goes with it.

7.7.12 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: Use your method from the first example to investigate pentagonal numbers.

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Grade 8:

Standard 1 — Number Sense

Understanding the number system is the basis of mathematics. Students extend their understanding of irrational numbers, such as π and the square root of 2, learning the relationship between the nature of the decimal of a number and whether it is rational or irrational. They use negative exponents to write decimals in scientific notation, and they use the inverse relationship between squaring and finding a square root to calculate approximate square roots.

Standard 2 — Computation

Fluency in computation is essential. Students add, subtract, multiply, and divide rational numbers. They use percentages to calculate simple and compound interest. They also use mental arithmetic to compute with fractions, decimals, powers, and percentages.

Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level write and solve linear equations and inequalities, including solving pairs of linear equations by the substitution method. They use properties of the rational numbers to evaluate and simplify algebraic expressions. They further extend their understanding of the relationship between equations and graphs by connecting slopes to rates of change and by drawing graphs of quadratic functions and simple cubic functions.

Standard 4 — Geometry

Students learn about geometric shapes and develop a sense of space. They learn new concepts relating to shapes, such as altitudes, bisectors, and chords and perform constructions connected with them. They further develop their sense of three-dimensional space by investigating how objects intersect in space. They draw a wide range of transformations of shapes, and they apply the Pythagorean Theorem and its converse to problems in two- and three-dimensions.

Standard 5 — Measurement

The study of measurement is essential because of its uses in many aspects of everyday life. Students convert common measurements for lengths, areas, volumes, weights, capacities, and times. They develop and use the concept of rate and derived measures — e.g., velocity and density. They apply the concepts of similarity, ratio, and proportion to problems involving scale factors, areas, and volumes. They find areas, perimeters, volumes, and surface areas, including those of irregular shapes made up of more basic shapes.

Standard 6 — Data Analysis and Probability

Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they evaluate whether claims based on data are reasonable and employ various sampling methods, analyzing their strengths and weaknesses. They understand the concepts of the median and quartiles and use these measures to draw and analyze box-and-whisker plots. They represent and analyze two-variable data using scatterplots. They understand the concept of equally likely events and use it to find probabilities. They also find the number of arrangements of objects using the Basic Counting Principle.

Standard 7 — Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with irrational numbers, analyzing graphs, or finding surface areas, for example, students move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Reasoning and Proof

Mathematics is developed by using known ideas and concepts to develop others. Repeated addition becomes multiplication. Multiplication of numbers less than ten can be extended to numbers less than one hundred and then to the entire number system. Knowing how to find the area of a right triangle extends to all right triangles. Extending patterns, finding even numbers, developing formulas, and proving the Pythagorean Theorem are all examples of mathematical reasoning. Students should learn to observe, generalize, make assumptions from known information, and test their assumptions.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, $\frac{1}{4}$, one divided by four, 0.25, $\frac{1}{8} + \frac{1}{8}$, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: $+$, $=$, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense

Students know the properties of rational and irrational* numbers expressed in a variety of forms. They understand and use exponents*, powers, and roots.*

8.1.1 Read, write, compare, and solve problems using decimals in scientific notation*.

Example: Write 0.00357 in scientific notation.

8.1.2 Know that every rational number is either a terminating or repeating decimal and that every irrational number is a nonrepeating decimal.

Example: Recognize that 2.375 is a terminating decimal, 5.121212… is a repeating decimal, and that $\pi = 3.14159265\ldots$ is a nonrepeating decimal. Name a rational number. Explain your reasoning.

8.1.3 Understand that computations with an irrational number and a rational number (other than zero) produce an irrational number.

Example: Tell whether the product of 7 and π is rational or irrational. Explain how you know that your answer is correct.

8.1.4 Understand and evaluate negative integer* exponents.

Example: Write 2^{-3} as a fraction.

8.1.5 Use the laws of exponents for integer exponents.

Example: Write $2^2 \times 2^3$ as $2 \times 2 \times 2 \times 2 \times 2$ and then as a single power of 2. Explain what you are doing.

8.1.6 Use the inverse relationship between squaring and finding the square root of a perfect square integer.

Example: Find the value of $(\sqrt{144})^2$.

8.1.7 Calculate and find approximations of square roots.

Example: For an integer that is not a perfect square, find the two integers (one larger, one smaller) that are closest to its square root and explain your reasoning.

* rational number: a real number that can be written as a ratio of two integers* (e.g., ½, ⅚, $^{23}⁄_{9}$)

* integer: …, -3, -2, -1, 0, 1, 2, 3, …

* irrational number: a real number that cannot be written as a ratio of two integers (e.g., π, $\sqrt{3}$, 7π)

* exponent: e.g., the exponent 4 in 3^4 tells you to write four 3s and compute $3 \times 3 \times 3 \times 3$

* scientific notation: a shorthand way of writing numbers using powers of ten (e.g., $300{,}000 = 3 \times 10^5$)

Standard 2

Computation

Students compute with rational numbers expressed in a variety of forms. They solve problems involving ratios, proportions, and percentages.*

8.2.1 Add, subtract, multiply, and divide rational numbers (integers*, fractions, and terminating decimals) in multi-step problems.

Example: $-3.4 + 2.8 \times 5.75 = ?$, $1\frac{4}{5} + -\frac{3}{8} \times 2\frac{2}{9} = ?$, $81.04 \div 17.4 - 2.79 = ?$.

8.2.2 Solve problems by computing simple and compound interest.

Example: You leave $100 in each of three bank accounts paying 5% interest per year. One account pays simple interest, one pays interest compounded annually, and the third pays interest compounded quarterly. Use a spreadsheet to find the amount of money in each account after one year, two years, three years, ten years, and twenty years. Compare the results in the three accounts and explain how compounding affects the balance in each account.

8.2.3 Use estimation techniques to decide whether answers to computations on a calculator are reasonable.

Example: Your friend uses his calculator to find 15% of $25 and gets $375. Without solving, explain why you think the answer is wrong.

8.2.4 Use mental arithmetic to compute with common fractions, decimals, powers, and percents.

Example: Find 20% of $50 without using pencil and paper.

8

* rational number: a real number that can be written as a ratio of two integers* (e.g., $\frac{1}{2}$, $\frac{5}{6}$, $\frac{23}{9}$)

* integer: …, -3, -2, -1, 0, 1, 2, 3, …

Standard 3

Algebra and Functions

Students solve simple linear equations and inequalities. They interpret and evaluate expressions involving integer powers. They graph and interpret functions. They understand the concepts of slope* and rate.*

8.3.1 Write and solve linear equations and inequalities in one variable, interpret the solution or solutions in their context, and verify the reasonableness of the results.

Example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be least $100. Write an inequality for the number of sales you need to make, solve it, and check that your answer is reasonable.

8.3.2 Solve systems of two linear equations using the substitution method and identify approximate solutions graphically.

Example: Solve the system. $2x + 3y = 7$
$x + 2y = 5$

8.3.3 Interpret positive integer powers as repeated multiplication and negative integer powers as repeated division or multiplication by the multiplicative inverse.

Example: Use a spreadsheet to explore the relationship between positive and negative integer powers by making a table of values of powers of 3, from 3^{-5} to 3^5.

8.3.4 Use the correct order of operations to find the values of algebraic expressions involving powers.

Example: Use a scientific calculator to find the value of $3(2x + 5)^2$ when $x = -35$.

8.3.5 Identify and graph linear functions and identify lines with positive and negative slope.

Example: Draw the graphs of $y = 2x - 1$, $y = 3x - 1$, $y = -2x - 1$, and $y = -3x - 1$. Find the slope of each graph. What do you notice?

8.3.6 Find the slope of a linear function given the equation and write the equation of a line given the slope and any point on the line.

Example: Write an equation of the line with slope 2 and y-intercept -4.

8.3.7 Demonstrate an understanding of rate as a measure of one quantity with respect to another quantity.

Example: A car moving at a constant speed travels 90 km in 2 hours, 135 km in 3 hours, 180 km in 4 hours, etc. Draw a graph of distance as a function of time and find the slope of the graph. Explain what the slope tells you about the movement of the car.

8.3.8 Demonstrate an understanding of the relationships among tables, equations, verbal expressions, and graphs of linear functions.

Example: Write an equation that represents the verbal description: "the perimeter of a square is four times the side length." Construct a table of values for this relationship and draw its graph.

8.3.9 Represent simple quadratic functions using verbal descriptions, tables, graphs, and formulas and translate among these representations.

Example: Draw the graph of $y = x^2$, $y = 2x^2$, and $y = 3x^2$. Describe their similarities and differences.

8.3.10 Graph functions of the form $y = nx^2$ and $y = nx^3$ and describe the similarities and differences in the graphs.

Example: Draw the graphs of $y = 2x^2$ and $y = 2x^3$. Explain which graph shows faster growth.

* integer: …, -3, -2, -1, 0, 1, 2, 3, …

* slope: between any two points on a line, the slope is the change in vertical distance divided by the change in horizontal distance ("rise" over "run")

Standard 4

Geometry

Students deepen their understanding of plane and solid geometric shapes and properties by constructing shapes that meet given conditions, by identifying attributes of shapes, and by applying geometric concepts to solve problems.

8.4.1 Identify and describe basic properties of geometric shapes: altitudes*, diagonals, angle and perpendicular bisectors*, central angles*, radii, diameters, and chords*.

Example: Describe a central angle of a circle in words and draw a diagram.

8.4.2 Perform simple constructions, such as bisectors of segments and angles, copies of segments and angles, and perpendicular segments. Describe and justify the constructions.

Example: Explain the procedures used to construct the three angle bisectors of a triangle.

8.4.3 Identify properties of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more figures intersect in a plane or in space.

Example: Find two lines in your classroom that are not parallel, yet do not meet.

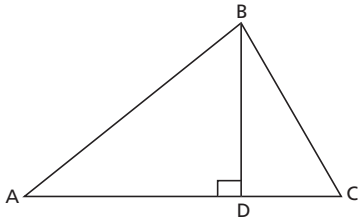
8.4.4 Draw the translation (slide), rotation (turn), reflection (flip), and dilation (stretches and shrinks) of shapes.

Example: Draw a rectangle and slide it 3 inches horizontally across your page. Then rotate it clockwise through 90º about the bottom left vertex. Draw the new rectangle in a different color.

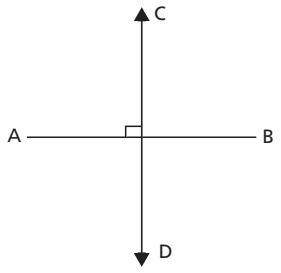
8.4.5 Use the Pythagorean Theorem and its converse to solve problems in two and three dimensions.

Example: Measure the dimensions of a shoe box and calculate the length of a diagonal from the top right to the bottom left of the box. Measure with a string to evaluate your solution.

* altitude: a line segment from the vertex of a triangle to meet the line containing the opposite side in a right angle (altitude is $\overline{BD}$ in $\triangle ABC$)

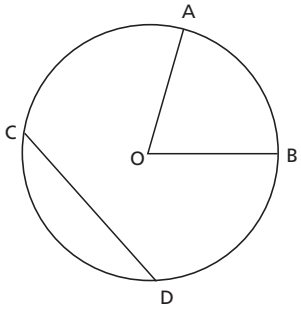


* perpendicular bisector: a line (or ray or line segment) at right angles to a given line segment that divides it in half ($\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$)



* central angle: the angle formed by joining two points on a circle to the center ($\angle AOB$ is a central angle)

* chord: a line segment joining two points on a circle ($\overline{CD}$ is a chord)



Standard 5

Measurement

Students convert between units of measure and use rates and scale factors to solve problems. They compute the perimeter, area, and volume of geometric objects. They investigate how perimeter, area, and volume are affected by changes of scale.

8.5.1 Convert common measurements for length, area, volume, weight, capacity, and time to equivalent measurements within the same system.

Example: The area of a hall is 40 square yards. What is the area in square feet?

8.5.2 Solve simple problems involving rates and derived measurements for attributes such as velocity and density.

Example: A car travels at 60 mph for 20 minutes. How far does it travel? What units are appropriate for distance? Explain your answer.

8.5.3 Solve problems involving scale factors, area, and volume using ratio and proportion.

Example: Calculate the volume and surface area of cubes with side 1 cm, 2 cm, 3 cm, etc. Make a table of your results and describe any patterns in the table.

8.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms*, trapezoids*, triangles, circles, prisms*, cylinders, spheres, cones, and pyramids.

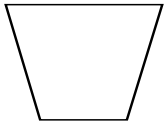
Example: Find the total surface area of a right triangular prism 14 feet high and with a base that measures 8 feet by 6 feet.

8.5.5 Estimate and compute the area of irregular two-dimensional shapes and the volume of irregular three-dimensional objects by breaking them down into more basic geometric objects.

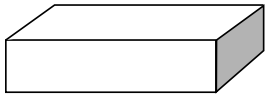
Example: Find the volume of a dog house that has a rectangular space that is 3 ft by 2 ft by 5 ft and has a triangular roof that is 1.5 ft higher than the walls of the house.

* parallelogram: a four-sided figure with both pairs of opposite sides parallel

* trapezoid: a four-sided figure with one pair of opposite sides parallel



* prism: a solid shape with fixed cross-section (a right prism is a solid shape with two parallel faces that are congruent polygons and other faces that are rectangles)



Standard 6

Data Analysis and Probability

Students collect, organize, represent, and interpret relationships in data sets that have one or more variables. They determine probabilities and use them to make predictions about events.

8.6.1 Identify claims based on statistical data and, in simple cases, evaluate the reasonableness of the claims. Design a study to investigate the claim.

Example: A study shows that teenagers who use a certain brand of toothpaste have fewer cavities than those using other brands. Describe how you can test this claim in your school.

8.6.2 Identify different methods of selecting samples, analyzing the strengths and weaknesses of each method, and the possible bias in a sample or display.

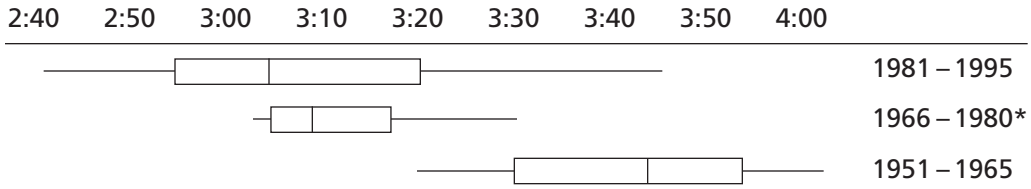
Example: Describe possible bias in the following survey: A local television station has a daily call-in poll. Viewers of the morning and noon newscasts are asked to call one telephone number to answer "yes" and a different telephone number to answer "no." The results are reported on the six-o'clock newscast.

8.6.3 Understand the meaning of, and be able to identify or compute the minimum value, the lower quartile*, the median*, the upper quartile*, the interquartile range, and the maximum value of a data set.

Example: Arrange a set of test scores in increasing order and find the lowest and highest scores, the median, and the upper and lower quartiles.

8.6.4 Analyze, interpret, and display single- and two-variable data in appropriate bar, line, and circle graphs; stem-and-leaf plots*; and box-and-whisker plots* and explain which types of display are appropriate for various data sets.

Example: The box-and-whisker plots below show winning times (hours:minutes) for the Indianapolis 500 race in selected years:



*Except 1967, 1973, 1975, and 1976.

In the years from 1951–1965, the slowest time was 3 h 57 min. Explain how the slowest time changed through the years 1951–1995. How did winning times change during that period? How did the median times change in the same period?

8.6.5 Represent two-variable data with a scatterplot* on the coordinate plane and describe how the data points are distributed. If the pattern appears to be linear, draw a line that appears to best fit the data and write the equation of that line.

Example: Survey some of the students at each grade level in your school, asking them how much time they spend on homework. Plot the grade level and time of each student as a point (grade, time) on a scatter diagram. Describe and justify any relationship between grade and time spent on homework.

8.6.6 Understand and recognize equally likely events.

Example: When you roll a number cube, what is the probability that the number on the top face will be a 6? Explain your answer.

8.6.7 **Find the number of possible arrangements of several objects by using the Basic Counting Principle.**

Example: You are planning to place four pictures in a line on a shelf. Find the number of ways you can arrange the four pictures.

* lower quartile: the value that separates the lowest one-fourth of the values from the rest of the values
* median: the value that divides a set of data, written in order of size, into two equal parts
* upper quartile: the value that separates the highest one-fourth of the values from the rest of the values
* stem-and-leaf plot: e.g., this one shows 62, 63, 67, 71, 75, 75, 76, etc.

Stem	Leaf
6	2 3 7
7	1 5 5 6 8 9
8	0 1 1 2 3 5 5 7 8 8
9	1 2 2 3 3 4

* box-and-whisker plot: a diagram showing median, quartiles, and range (see diagram on previous page)
* scatterplot: a coordinate graph showing ordered pairs of data

8

Standard 7

Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

8.7.1 **Analyze problems by identifying relationships, telling relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.**

Example: Solve the problem: "For computers, binary numbers are great because they are simple to work with and they use just two values of voltage, magnetism, or other signal. This makes hardware easier to design and more noise resistant. Binary numbers let you represent any amount you want using just two digits: 0 and 1. The number you get when you count ten objects is written 1010. In expanded notation, this is $1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$. Write the number for thirteen in the binary (base 2) system." Decide to make an organized list.

8.7.2 **Make and justify mathematical conjectures based on a general description of a mathematical question or problem.**

Example: In the first example, if you have only two symbols, 0 and 1, then one object: 1, two objects: 10, three objects: 11, four objects: 100. Predict the symbol for five objects.

8.7.3 **Decide when and how to divide a problem into simpler parts.**

Example: In the first example, write expanded notation for the number five in base 2; begin with the fact that $5 = 4 + 1$.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

8.7.4 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, write the first five numbers in base 2 notation and look for a pattern.

8.7.5 Make and test conjectures using inductive reasoning.

Example: In the first example, predict the base 2 notation for six objects, then use expanded notation to test your prediction.

8.7.6 Express solutions clearly and logically using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, explain how you will find the base two notation for thirteen objects.

8.7.7 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: Measure the length and width of a basketball court. Use the Pythagorean Theorem to calculate the length of a diagonal. How accurately should you give your answer?

8.7.8 Select and apply appropriate methods for estimating results of rational-number computations.

Example: Use a calculator to find the cube of 15. Check your answer by finding the cubes of 10 and 20.

8

8.7.9 Use graphing to estimate solutions and check the estimates with analytic approaches.

Example: Use a graphing calculator to draw the straight line $x + y = 10$. Use this to estimate solutions of the inequality $x + y > 10$ by testing points on each side of the line.

8.7.10 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, list the first thirteen numbers in base 2 notation. Use patterns or expanded notation to confirm your list.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

8.7.11 Decide whether a solution is reasonable in the context of the original situation.

Example: In the basketball court example, does the accuracy of your answer depend on your initial measuring?

8.7.12 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: In the first example, use your list of base 2 numbers to add numbers in base 2. Explain exactly how your addition process works.

Algebra I

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Algebra I:

Standard 1 — Operations With Real Numbers

Students deepen their understanding of real numbers by comparing expressions involving square roots and exponents. They use the properties of real numbers to simplify algebraic formulas, and they convert between different measurement units using dimensional analysis.

Standard 2 — Linear Equations and Inequalities

Students solve linear equations to find the value of the variable and they rearrange formulas. They solve linear inequalities by using order properties of the real numbers, and they solve word problems involving linear equations, inequalities, and formulas.

Standard 3 — Relations and Functions

Students draw and interpret graphs of relations. They understand the concept of a function, find domains and ranges, and link equations to functions.

Standard 4 — Graphing Linear Equations and Inequalities

Students draw graphs of straight lines and relate their equations to their slopes and intercepts. They model situations with linear equations and use them to make predictions, and they graph linear inequalities in two variables.

Standard 5 — Pairs of Linear Equations and Inequalities

Students solve pairs of linear equations in two variables using both graphs and algebraic methods. They use pairs of linear equations to solve word problems, and they use graphs to solve pairs of linear inequalities in two variables.

Standard 6 — Polynomials

Students operate with polynomials, adding, subtracting, multiplying, dividing, and raising to powers. They find factors of polynomials, learning special techniques for factoring quadratics. They understand the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial.

Standard 7 — Algebraic Fractions

Students simplify algebraic fractions, using what they have learned about factoring polynomials. They solve algebraic proportions.

Standard 8 — Quadratic, Cubic, and Radical Equations

Students draw graphs of quadratic, cubic, and rational functions. They derive the formula for solving quadratic equations and solve these equations by using the formula, by factoring, and by completing the square. They also solve equations that contain radical expressions and use graphing calculators to find approximate solutions of equations.

Standard 9 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn about inductive and deductive reasoning and how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

A1

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Operations With Real Numbers

Students simplify and compare expressions. They use rational exponents and simplify square roots.

A1.1.1 Compare real number expressions.

Example: Which is larger: 2^3 or $\sqrt{49}$?

A1.1.2 Simplify square roots using factors.

Example: Explain why $\sqrt{48} = 4\sqrt{3}$.

A1.1.3 Understand and use the distributive, associative, and commutative properties.

Example: Simplify $(6x^2 - 5x + 1) - 2(x^2 + 3x - 4)$ by removing the parentheses and rearranging. Explain why you can carry out each step.

A1.1.4 Use the laws of exponents for rational exponents.

Example: Simplify $25^{3/2}$.

A1.1.5 Use dimensional (unit) analysis to organize conversions and computations.

Example: Convert 5 miles per hour to feet per second: $\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \approx 7.3 \text{ ft/sec}$.

Standard 2

Linear Equations and Inequalities

Students solve linear equations and inequalities in one variable. They solve word problems that involve linear equations, inequalities, or formulas.

A1.2.1 Solve linear equations.

Example: Solve the equation $7a + 2 = 5a - 3a + 8$.

A1.2.2 Solve equations and formulas for a specified variable.

Example: Solve the equation $q = 4p - 11$ for p.

A1.2.3 Find solution sets of linear inequalities when possible numbers are given for the variable.

Example: Solve the inequality $6x - 3 > 10$ for x in the set $\{0, 1, 2, 3, 4\}$.

A1.2.4 Solve linear inequalities using properties of order.

Example: Solve the inequality $8x - 7 \leq 2x + 5$, explaining each step in your solution.

A1.2.5 Solve combined linear inequalities.

Example: Solve the inequalities $-7 < 3x + 5 < 11$.

A1.2.6 Solve word problems that involve linear equations, formulas, and inequalities.

Example: You are selling tickets for a play that cost $3 each. You want to sell at least $50 worth. Write and solve an inequality for the number of tickets you must sell.

Standard 3

Relations and Functions

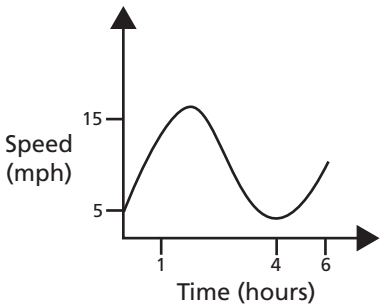
Students sketch and interpret graphs representing given situations. They understand the concept of a function and analyze the graphs of functions.

A1.3.1 Sketch a reasonable graph for a given relationship.

Example: Sketch a reasonable graph for a person's height from age 0 to 25.

A1.3.2 Interpret a graph representing a given situation.

Example: Jessica is riding a bicycle. The graph below shows her speed as it relates to the time she has spent riding. Describe what might have happened to account for such a graph.



A1.3.3 Understand the concept of a function, decide if a given relation is a function, and link equations to functions.

Example: Use either paper or a spreadsheet to generate a list of values for x and y in $y = x^2$. Based on your data, make a conjecture about whether or not this relation is a function. Explain your reasoning.

A1.3.4 Find the domain and range of a relation.

Example: Based on the list of values from the last example, what are the domain and range of $y = x^2$?

Standard 4

Graphing Linear Equations and Inequalities

Students graph linear equations and inequalities in two variables. They write equations of lines and find and use the slope and y-intercept of lines. They use linear equations to model real data.

A1.4.1 Graph a linear equation.

Example: Graph the equation $3x - y = 2$.

A1.4.2 Find the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

Example: Find the slope and y-intercept of the line $4x + 6y = 12$.

A1.4.3 Write the equation of a line in slope-intercept form. Understand how the slope and y-intercept of the graph are related to the equation.

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

A1.4.4 **Write the equation of a line given appropriate information.**

Example: Find an equation of the line through the points (1, 4) and (3, 10), then find an equation of the line through the point (1, 4) perpendicular to the first line.

A1.4.5 **Write the equation of a line that models a data set and use the equation (or the graph of the equation) to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.**

Example: As your family is traveling along an interstate, you note the distance traveled every 5 minutes. A graph of time and distance shows that the relation is approximately linear. Write the equation of the line that fits your data. Predict the time for a journey of 50 miles. What does the slope represent?

A1.4.6 **Graph a linear inequality in two variables.**

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

Standard 5

Pairs of Linear Equations and Inequalities

Students solve pairs of linear equations using graphs and using algebra. They solve pairs of linear inequalities using graphs. They solve word problems involving pairs of linear equations.

A1.5.1 **Use a graph to estimate the solution of a pair of linear equations in two variables.**

Example: Graph the equations $3y - x = 0$ and $2x + 4y = 15$ to find where the lines intersect.

A1.5.2 **Use a graph to find the solution set of a pair of linear inequalities in two variables.**

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

A1.5.3 **Understand and use the substitution method to solve a pair of linear equations in two variables.**

Example: Solve the equations $y = 2x$ and $2x + 3y = 12$ by substitution.

A1.5.4 **Understand and use the addition or subtraction method to solve a pair of linear equations in two variables.**

Example: Use subtraction to solve the equations: $3x + 4y = 11$ and $3x + 2y = 7$.

A1.5.5 **Understand and use multiplication with the addition or subtraction method to solve a pair of linear equations in two variables.**

Example: Use multiplication with the subtraction method to solve the equations: $x + 4y = 16$ and $3x + 2y = -3$.

A1.5.6 **Use pairs of linear equations to solve word problems.**

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$ and the expenses for that product can be represented by the equation $y = 5.25x + 10{,}000$, where x is the amount of the product sold and y is the number of dollars. How much of the product must be sold for the company to reach the break-even point?

Standard 6

Polynomials

Students add, subtract, multiply, and divide polynomials. They factor quadratics.

A1.6.1 Add and subtract polynomials.

Example: Simplify $(4x^2 - 7x + 2) - (x^2 + 4x - 5)$.

A1.6.2 Multiply and divide monomials.

Example: Simplify $a^2b^5 \div ab^2$.

A1.6.3 Find powers and roots of monomials (only when the answer has an integer exponent).

Example: Find the square root of a^2b^6.

A1.6.4 Multiply polynomials.

Example: Multiply $(n + 2)(4n - 5)$.

A1.6.5 Divide polynomials by monomials.

Example: Divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.

A1.6.6 Find a common monomial factor in a polynomial.

Example: Factor $36xy^2 + 18xy^4 - 12x^2y^4$.

A1.6.7 Factor the difference of two squares and other quadratics.

Example: Factor $4x^2 - 25$ and $2x^2 - 7x + 3$.

A1.6.8 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x-intercepts of $y = 3x^2 - 5x - 1$ as the solutions of the equation.

Standard 7

Algebraic Fractions

Students simplify algebraic ratios and solve algebraic proportions.

A1.7.1 Simplify algebraic ratios.

Example: Simplify $\frac{x^2 - 16}{x^2 + 4x}$.

A1.7.2 Solve algebraic proportions.

Example: Create a tutorial to be posted to the school's Web site to instruct beginning students in the steps involved in solving an algebraic proportion. Use $\frac{x + 5}{4} = \frac{3x + 5}{7}$ as an example.

Standard 8

Quadratic, Cubic, and Radical Equations

Students graph and solve quadratic and radical equations. They graph cubic equations.

A1.8.1 Graph quadratic, cubic, and radical equations.

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet (generate a data set), display the graph to check your work.

A1.8.2 Solve quadratic equations by factoring.

Example: Solve the equation $x^2 - 3x + 2 = 0$ by factoring.

A1.8.3 Solve quadratic equations in which a perfect square equals a constant.

Example: Solve the equation $(x - 7)^2 = 64$.

A1.8.4 Complete the square to solve quadratic equations.

Example: Solve the equation $x^2 - 7x + 9 = 0$ by completing the square.

A1.8.5 Derive the quadratic formula by completing the square.

Example: Prove that the equation $ax^2 + bx + c = 0$ has solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

A1.8.6 Solve quadratic equations using the quadratic formula.

Example: Solve the equation $x^2 - 7x + 9 = 0$.

A1.8.7 Use quadratic equations to solve word problems.

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

A1.8.8 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x + 6} = x$.

A1.8.9 Use graphing technology to find approximate solutions of quadratic and cubic equations.

Example: Use a graphing calculator to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth.

Standard 9

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

A1.9.1 Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: Fran has scored 16, 23, and 30 points in her last three games. How many points must she score in the next game so that her four-game average does not fall below 20 points?

A1.9.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 10 points. Is his answer reasonable? Why or why not?

Students develop and evaluate mathematical arguments and proofs.

A1.9.3 Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Given an argument (such as $3x + 7 > 5x + 1$, and therefore $-2x > -6$, and therefore $x > 3$), provide a visual presentation of a step-by-step check, highlighting any errors in the argument.

A1.9.4 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: Try "solving" the equations $x + 3y = 5$ and $5x + 15y = 25$ simultaneously. Explain what went wrong.

A1.9.5 Decide whether a given algebraic statement is true always, sometimes, or never (statements involving linear or quadratic expressions, equations, or inequalities).

Example: Is the statement $x^2 - 5x + 2 = x^2 + 5x + 2$ true for all x, for some x, or for no x? Explain your answer.

A1.9.6 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

Example: What type of reasoning are you using when you look for a pattern?

A1.9.7 Identify the hypothesis and conclusion in a logical deduction.

Example: What is the hypothesis and conclusion in this argument: If there is a number x such that $2x + 1 = 7$, then $x = 3$?

A1.9.8 Use counterexamples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

Example: Use the demonstration-graphing calculator on an overhead projector to produce an example showing that this statement is false: all quadratic equations have two different solutions.

Algebra II

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Algebra II:

Standard 1 — Relations and Functions

Students recognize and graph polynomial, rational, and algebraic functions. They understand the concept of functional notation and use it to combine functions by composition. They solve equations and inequalities by examining their graphs and interpret situations as functions in graphs, formulas, and words.

Standard 2 — Linear and Absolute Value Equations and Inequalities

Students graph linear equations and inequalities involving absolute value. They use a variety of methods to solve systems of up to three linear equations in up to three variables, and they model data with linear equations and make predictions from the results.

Standard 3 — Quadratic Equations and Functions

Students extend the number system by defining complex numbers, relating them to the real numbers, and using them to solve quadratic equations. They draw graphs of quadratic functions and apply transformations to the functions. They find and interpret zeros and maximum and minimum values, and solve word problems. They also solve equations containing radicals and solve pairs of equations.

Standard 4 — Conic Sections

Students write equations and draw graphs of conic sections (circle, ellipse, parabola, and hyperbola), thus relating an algebraic representation to a geometric one.

Standard 5 — Polynomials

Students understand and use the binomial theorem for positive integer powers. They learn techniques for factoring polynomials in order to solve equations and related word problems. They find approximate solutions of equations using graphing technology and write equations with given solutions. They understand the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial.

Standard 6 — Algebraic Fractions

Students understand and use the concepts of negative and fractional exponents. They add, subtract, multiply, divide, and simplify algebraic fractions. They solve equations involving algebraic fractions and solve related word problems. They also solve problems of direct, inverse, and joint variation.

Standard 7 — Logarithmic and Exponential Functions

Students understand the concepts of logarithmic and exponential functions. They graph exponential functions and solve problems of growth and decay. They understand the inverse relationship between exponents and logarithms and use it to prove laws of logarithms and to solve equations. And they convert logarithms between bases and simplify logarithmic expressions.

Standard 8 — Sequences and Series

Students define the concepts of arithmetic and geometric sequences and series. They find specified terms of sequences and partial sums of series and use their knowledge of sequences and series to solve word problems.

Standard 9 — Counting Principles and Probability

Students understand and apply counting principles to find permutations and combinations and related probabilities.

Standard 10 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Relations and Functions

Students graph relations and functions and find zeros. They use function notation and combine functions by composition. They interpret functions in given situations.

A2.1.1 Recognize and graph various types of functions, including polynomial, rational, and algebraic functions.

Example: Draw the graphs of the functions $y = x^4 - x^2$, $y = \frac{7}{x-2}$, and $y = \sqrt{x+2}$.

A2.1.2 Use function notation. Add, subtract, multiply, and divide pairs of functions.

Example: Let $f(x) = 7x + 2$ and $g(x) = x^2$. Find the value of $f(x) \bullet g(x)$.

A2.1.3 Understand composition of functions and combine functions by composition.

Example: Let $f(x) = x^3$ and $g(x) = x - 2$. Find $f(g(x))$.

A2.1.4 Graph relations and functions with and without graphing technology.

Example: Draw the graph of $y = x^3 - 3x^2 - x + 3$.

A2.1.5 Find the zeros of a function.

Example: In the last example, find the zeros of the function; i.e., find x when $y = 0$.

A2.1.6 Solve an inequality by examining the graph.

Example: Find the solution for $x^3 - 3x^2 - x + 3 < 0$ by graphing $y = x^3 - 3x^2 - x + 3$.

A2.1.7 Graph functions defined piece-wise.

Example: Sketch the graph of $f(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x > 0 \end{cases}$.

A2.1.8 Interpret given situations as functions in graphs, formulas, and words.

Example: You and your parents are going to Boston and want to rent a car at Logan International Airport on a Monday morning and drop the car off in downtown Providence, R.I., on the following Wednesday. Find the rates from two national car companies and plot the costs on a graph. Decide which company offers the best deal. Explain your answer.

Standard 2

Linear and Absolute Value Equations and Inequalities

Students solve systems of linear equations and inequalities and use them to solve word problems. They model data with linear equations.

A2.2.1 Graph absolute value equations and inequalities.

Example: Draw the graph of $y = 2x - 5$ and use that graph to draw the graph of $y = |2x - 5|$.

A2.2.2 Use substitution, elimination, and matrices to solve systems of two or three linear equations in two or three variables.

Example: Solve the system of equations: $x - 2y + 3z = 5, x + 3z = 11, 5y - 6z = 9$.

A2.2.3 Use systems of linear equations and inequalities to solve word problems.

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

A2.2.4 Find a linear equation that models a data set using the median fit method and use the model to make predictions.

Example: You light a candle and record its height in centimeters every minute. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.5), (3, 14.8), (4, 13.2), (5, 11.5), (6, 10.0), (7, 8.2), (9, 4.9), and (10, 3.1). Find the median fit line to express the candle's height as a function of the time and state the meaning of the slope in terms of the burning candle.

Standard 3

Quadratic Equations and Functions

Students solve quadratic equations, including the use of complex numbers. They interpret maximum and minimum values of quadratic functions. They solve equations that contain square roots.

A2.3.1 Define complex numbers and perform basic operations with them.

Example: Multiply $7 - 4i$ and $10 + 6i$.

A2.3.2 Understand how real and complex numbers are related, including plotting complex numbers as points in the plane.

Example: Plot the points corresponding to $3 - 2i$ and $1 + 4i$. Add these complex numbers and plot the result. How is this point related to the other two?

A2.3.3 Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

A2.3.4 Graph quadratic functions. Apply transformations to quadratic functions. Find and interpret the zeros and maximum or minimum value of quadratic functions.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

A2.3.5 Solve word problems using quadratic equations.

Example: You have 100 feet of fencing to make three sides of a rectangular area using an existing straight fence as the fourth side. Construct a formula in a spreadsheet to determine the area you can enclose and use the spreadsheet to make a conjecture about the maximum area possible. Prove (or disprove) your conjecture by solving an appropriate quadratic equation.

A2.3.6 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x+9} = 9 - \sqrt{x}$.

A2.3.7 Solve pairs of equations, one quadratic and one linear or both quadratic.

Example: Solve the system of equations $y = x^2 - 5x + 1, x + y + 2 = 0$.

Standard 4

Conic Sections

Students write equations of conic sections and draw their graphs.

A2.4.1 Write the equations of conic sections (circle, ellipse, parabola, and hyperbola).

Example: Write an equation for a parabola with focus $(2, 3)$ and directrix $y = 1$.

A2.4.2 Graph conic sections.

Example: Graph the circle described by the equation $(x + 4)^2 + (y - 1)^2 = 9$.

Standard 5

Polynomials

Students use the binomial theorem, divide and factor polynomials, and solve polynomial equations.

A2.5.1 Understand the binomial theorem and use it to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$.

A2.5.2 Divide polynomials by others of lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

A2.5.3 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

A2.5.4 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

A2.5.5 Use polynomial equations to solve word problems.

Example: You want to make an open-top box with a volume of 500 square inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Find the possible dimensions of the box.

A2.5.6 Write a polynomial equation given its solutions.

Example: Write an equation that has solutions $x = 2, x = 5i$ and $x = -5i$.

A2.5.7 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

Standard 6

Algebraic Fractions

Students use negative and fractional exponents. They simplify algebraic fractions and solve equations involving algebraic fractions. They solve problems of direct, inverse, and joint variation.

A2.6.1 Understand and use negative and fractional exponents.

Example: Simplify $(2a^{-2}b^3)^4 (4a^3b^{-1})^{-2}$.

A2.6.2 Add, subtract, multiply, divide, and simplify algebraic fractions.

Example: Simplify $\frac{x^2 - 4}{x^5} \div \frac{x^3 - 8}{x^8}$.

A2.6.3 Simplify complex fractions.

Example: Simplify $\left(\frac{5}{x-2} + \frac{2}{x+3}\right) \div \left(\frac{1}{x+3} + \frac{7}{x-2}\right)$.

A2.6.4 Solve equations involving algebraic fractions.

Example: Solve $\frac{10}{n} + \frac{5}{n^2 - 4} = \frac{7}{n - 2}$.

A2.6.5 Solve word problems involving fractional equations.

Example: Two students, working independently, can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together?

A2.6.6 Solve problems of direct, inverse, and joint variation.

Example: One day your drive to work takes 10 minutes and you average 30 mph. The next day the drive takes 15 minutes. What is your average speed that day?

Standard 7

Logarithmic and Exponential Functions

Students graph exponential functions and relate them to logarithms. They solve logarithmic and exponential equations and inequalities. They solve word problems using exponential functions.

A2.7.1 Graph exponential functions.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

A2.7.2 Prove simple laws of logarithms.

Example: Use the fact that $a^x \bullet a^y = a^{x+y}$ to show that $\log_a(pq) = \log_a p + \log_a q$.

A2.7.3 Understand and use the inverse relationship between exponents and logarithms.

Example: Find the value of $\log_{10}(10^7)$.

A2.7.4 Solve logarithmic and exponential equations and inequalities.

Example: Solve the equation $\log_2 x = 5$.

A2.7.5 Use the definition of logarithms to convert logarithms from one base to another.

Example: Write $\log_{10} 75$ as a logarithm to base 2.

A2.7.6 Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values.

Example: Simplify $\log_3 81$.

A2.7.7 Use calculators to find decimal approximations of natural and common logarithmic numeric expressions.

Example: Find a decimal approximation for $\ln 500$.

A2.7.8 Solve word problems involving applications of exponential functions to growth and decay.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

A2

Standard 8

Sequences and Series

Students define and use arithmetic and geometric sequences and series.

A2.8.1 Define arithmetic and geometric sequences and series.

Example: What type of sequence is 10, 100, 1,000, 10,000, …?

A2.8.2 Find specified terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15, … .

A2.8.3 Find partial sums of arithmetic and geometric series.

Example: In the last example, find the sum of the first 10 terms.

A2.8.4 Solve word problems involving applications of sequences and series.

Example: You have on a Petri dish 1 square millimeter of a mold that doubles in size each day. What area will it cover after a month?

Standard 9

Counting Principles and Probability

Students use fundamental counting principles to compute combinations, permutations, and probabilities.

A2.9.1 Understand and apply counting principles to compute combinations and permutations.

Example: There are 5 students who work in a bookshop. If the bookshop needs 3 people to operate, how many days straight could the bookstore operate without the same group of students working twice?

A2.9.2 Use the basic counting principle, combinations, and permutations to compute probabilities.

Example: You are on a chess team made up of 15 players. What is the probability that you will be chosen if a 3-person team is selected at random?

Standard 10

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

A2.10.1 Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: The swimming pool at Roanoke Park is 24 feet long and 18 feet wide. The park district has determined that they have enough money to put a walkway of uniform width, with a maximum area of 288 square feet, around the pool. How could you find the maximum width of a new walkway?

A2.10.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 20 feet. Is that reasonable?

Students develop and evaluate mathematical arguments and proofs.

A2.10.3 Decide if a given algebraic statement is true always, sometimes, or never (statements involving rational or radical expressions or logarithmic or exponential functions).

Example: Is the statement $(a^x)^y = a^{xy}$ true for all x, for some x, or for no x?

A2.10.4 Use the properties of number systems and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Simplify $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$, explaining why you can take each step.

A2.10.5 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: A student solving the equation $\sqrt{x + 6} = x$ comes up with the solution set $\{-2, 3\}$. Explain why $\{-2, 3\}$ is not the solution set to this equation, and why the "check" step is essential in solving the equation.

A2.10.6 Use counterexamples to show that statements are false.

Example: Show by an example that this statement is false: The product of two complex numbers is never a real number.

NOTES

Calculus, Advanced Placement

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Calculus, Advanced Placement:

Standard 1 — Limits and Continuity

Students develop an understanding of the concept of limit by estimating limits from graphs and tables of values, and finding limits by substitution, and factoring rational functions. They extend the idea of a limit to one-sided limits and limits at infinity. They use limits to define and understand the concept of continuity, decide whether a function is continuous at a point, and find types of discontinuities. And they understand and apply two continuity theorems: the Intermediate Value Theorem and the Extreme Value Theorem.

Standard 2 — Differential Calculus

Students develop an understanding of the derivative as an instantaneous rate of change, using geometrical, numerical, and analytical methods. They use this definition to find derivatives of many types of functions and combinations of these functions (using, for example, sums, composites, and inverses). They also find second and higher order derivatives. They understand and use the relationship between differentiability and continuity. They understand and apply the Mean Value Theorem.

Standard 3 — Applications of Derivatives

Students apply what they learn about derivatives to finding slopes of curves and the related tangent lines. They analyze and graph functions, finding where they are increasing or decreasing, their maximum and minimum points, their points of inflection, and their concavity. They solve optimization problems, find average and instantaneous rates of change (including velocities and accelerations), and model rates of change.

Standard 4 — Integral Calculus

Students understand that integration is used to find areas and they evaluate integrals using rectangular approximations. From this, they develop the idea that integration is the inverse operation to differentiation — the Fundamental Theorem of Calculus. They use this result to find definite and indefinite integrals, including using the method of integration by substitution. They also apply approximate methods, such as the Trapezoidal Rule, to find definite integrals.

Standard 5 — Applications of Integration

Students apply what they learn about integrals to finding velocities from accelerations, solving separable differential equations, and finding areas and volumes. They also apply integration to model and solve problems in physics, biology, economics, etc.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to investigating limits and applying them to continuity, differentiability, and integration.

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: $+$, $=$, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

C

Standard 1

Limits and Continuity

Students understand the concept of limit, find limits of functions at points and at infinity, decide if a function is continuous, and use continuity theorems.

C.1.1 Understand the concept of limit and estimate limits from graphs and tables of values.

Example: Estimate $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$ by calculating the function's values for $x = 2.1, 2.01, 2.001$ and for $x = 1.9, 1.99, 1.999$.

C.1.2 Find limits by substitution.

Example: Find $\lim_{x \to 5} (2x + 1)$.

C.1.3 Find limits of sums, differences, products, and quotients.

Example: Find $\lim_{x \to \pi} (\sin x \bullet \cos x + \tan x)$.

C.1.4 Find limits of rational functions that are undefined at a point.

Example: Find $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$ by factoring and canceling.

C.1.5 Find one-sided limits.

Example: Find $\lim_{x \to 4} -\sqrt{4 - x}$.

C.1.6 Find limits at infinity.

Example: Find $\lim_{x \to \infty} \frac{x}{x - 1}$.

C.1.7 Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior.

Example: Find $\lim_{x \to 0} \frac{1}{x^2}$.

C.1.8 Find special limits such as $\lim_{x \to 0} \frac{\sin x}{x}$.

Example: Use a diagram to show that the limit above is equal to 1.

C.1.9 Understand continuity in terms of limits.

Example: Show that $f(x) = 3x + 1$ is continuous at $x = 2$ by finding $\lim_{x \to 2} (3x + 1)$ and comparing it with $f(2)$.

C.1.10 Decide if a function is continuous at a point.

Example: Show that $f(x) = \frac{x^2 + 2x - 8}{x - 2}$ is continuous at $x = 2$, provided that you define $f(2) = 6$.

C.1.11 Find the types of discontinuities of a function.

Example: What types of discontinuities has $h(x) = \frac{x^2 - 5x + 6}{x^2 - 4}$? Explain your answer.

C.1.12 Understand and use the Intermediate Value Theorem on a function over a closed interval.

Example: Show that $g(x) = 3 - x^2$ has a zero between $x = 1$ and $x = 2$, because it is continuous.

C.1.13 Understand and apply the Extreme Value Theorem: If $f(x)$ is continuous over a closed interval, then f has a maximum and a minimum on the interval.

Example: Decide if $t(x) = \tan x$ has a maximum value over the interval $[-\pi/4, \pi/4]$. What about the interval $[-\pi, \pi]$? Explain your answers.

C

Standard 2

Differential Calculus

Students find derivatives of algebraic, trigonometric, logarithmic, and exponential functions. They find derivatives of sums, products, and quotients, and composite and inverse functions. They find derivatives of higher order and use logarithmic differentiation and the Mean Value Theorem.

C.2.1 Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change.

Example: Find the derivative of $f(x) = x^2$ at $x = 5$ by calculating values of $\frac{x^2 - 5^2}{x - 5}$ for x near 5. Use a diagram to explain what you are doing and what the result means.

C.2.2 State, understand, and apply the definition of derivative.

Example: Find $\lim_{x \to 5} \frac{x^2 - 5^2}{x - 5}$. What does the result tell you?

C.2.3 Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions.

Example: Find dy/dx for the function $y = x^5$.

C.2.4 Find the derivatives of sums, products, and quotients.

Example: Find the derivative of $x \cos x$.

C.2.5 Find the derivatives of composite functions, using the chain rule.

Example: Find $f'(x)$ for $f(x) = (x^2 + 2)^4$.

C.2.6 Find the derivatives of implicitly-defined functions.

Example: For $xy - x^2y^2 = 5$, find dy/dx at the point $(2, 3)$.

C.2.7 Find derivatives of inverse functions.

Example: Let $f(x) = 2x^3$ and $g = f^{-1}$. Find $g'(2)$.

C.2.8 Find second derivatives and derivatives of higher order.

Example: Find the second derivative of e^{5x}.

C.2.9 Find derivatives using logarithmic differentiation.

Example: Find dy/dx for $y = \sqrt{(x + 3)^3 (x - 7)}$.

C.2.10 Understand and use the relationship between differentiability and continuity.

Example: Is $f(x) = 1/x$ continuous at $x = 0$? Is $f(x)$ differentiable at $x = 0$? Explain your answers.

C.2.11 Understand and apply the Mean Value Theorem.

Example: For $f(x) = \sqrt{x}$ on the interval $[1, 9]$, find the value of c such that $\frac{f(9) - f(1)}{9 - 1} = f'(c)$.

C

Standard 3

Applications of Derivatives

Students find slopes and tangents, maximum and minimum points, and points of inflection. They solve optimization problems and find rates of change.

C.3.1 **Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents.**

Example: Find the slope of the tangent to $y = x^3$ at the point $(2, 8)$.

C.3.2 **Find a tangent line to a curve at a point and a local linear approximation.**

Example: In the last example, find an equation of the tangent at $(2, 8)$.

C.3.3 **Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f'.**

Example: Use values of the derivative to find where $f(x) = x^3 - 3x$ is decreasing.

C.3.4 **Find local and absolute maximum and minimum points.**

Example: In the last example, find the local maximum and minimum points of $f(x)$.

C.3.5 **Analyze curves, including the notions of monotonicity and concavity.**

Example: In the last example, for which values of x is $f(x)$ decreasing and for which values of x is $f(x)$ concave upward?

C.3.6 **Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f''. Understand points of inflection as places where concavity changes.**

Example: In the last example, find the points of inflection of $f(x)$ and where $f(x)$ is concave upward and concave downward.

C.3.7 **Use first and second derivatives to help sketch graphs. Compare the corresponding characteristics of the graphs of f, f', and f''.**

Example: Use the last examples to draw the graph of $f(x) = x^3 - 3x$.

C.3.8 **Use implicit differentiation to find the derivative of an inverse function.**

Example: Let $f(x) = 2x^3$ and $g = f^{-1}$. Find $g'(x)$ using implicit differentiation.

C.3.9 **Solve optimization problems.**

Example: You want to enclose a rectangular area of 5,000 m^2. Find the shortest length of fencing you can use.

C.3.10 **Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed, and acceleration.**

Example: You are filling a bucket with water and the height H cm of the water after t seconds is given by $H(t) = (4t)^{2/3}$. How fast is the water rising 30 seconds after you start filling the bucket? Explain your answer.

C.3.11 **Find the velocity and acceleration of a particle moving in a straight line.**

Example: A bead on a wire moves so that, after t seconds, its distance s cm from the midpoint of the wire is given by $s = 5 \sin (t - \pi/4)$. Find its maximum velocity and where along the wire this occurs.

C.3.12 **Model rates of change, including related rates problems.**

Example: A boat is heading south at 10 mph. Another boat is heading west at 15 mph toward the same point. At these speeds, they will collide. Find the rate that the distance between them is decreasing 1 hour before they collide.

C

Standard 4

Integral Calculus

Students define integrals using Riemann Sums, use the Fundamental Theorem of Calculus to find integrals, and use basic properties of integrals. They integrate by substitution and find approximate integrals.

C.4.1 Use rectangle approximations to find approximate values of integrals.

Example: Find an approximate value for $\int_0^3 x^2 dx$ using 6 rectangles of equal width under the graph of $f(x) = x^2$.

C.4.2 Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points.

Example: Find the value of the Riemann Sum over the interval $[0, 3]$ using 6 subintervals of equal width for $f(x) = x^2$ evaluated at the midpoint of each subinterval.

C.4.3 Interpret a definite integral as a limit of Riemann Sums.

Example: Find the values of the Riemann Sums over the interval $[0, 3]$ using 12, 24, etc., subintervals of equal width for $f(x) = x^2$ evaluated at the midpoint of each subinterval. Find the limit of the Riemann Sums.

C.4.4 Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is $\int_a^b f'(x)dx = f(b) - f(a)$.

Example: Explain why $\int_4^5 2x dx = 5^2 - 4^2$.

C.4.5 Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.

Example: Using antiderivatives, find $\int_0^3 x^2 dx$.

C.4.6 Understand and use these properties of definite integrals:

$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

$\int_a^b k \bullet f(x)dx = k\int_a^b f(x)dx$

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

If $f(x) \le g(x)$ on $[a, b]$, then $\int_a^b f(x)dx \le \int_a^b g(x)dx$.

Example: Find $\int_0^3 5x^2 dx$, given that $\int_0^3 x^2 dx = 9$.

C.4.7 Understand and use integration by substitution (or change of variable) to find values of integrals.

Example: Find $\int_1^2 x^2(x^3 + 1)^4 dx$.

C.4.8 Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.

Example: Use the Trapezoidal Rule with 6 subintervals over $[0, 3]$ for $f(x) = x^2$ to approximate the value of $\int_0^3 x^2 dx$.

C

Standard 5

Applications of Integration

Students find velocity functions and position functions from their derivatives, solve separable differential equations, and use definite integrals to find areas and volumes.

C.5.1 Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line.

Example: A bead on a wire moves so that its velocity, after t seconds, is given by $v(t) = 3\cos 3t$. Given that it starts 2 cm to the left of the midpoint of the wire, find its position after 5 seconds.

C.5.2 Solve separable differential equations and use them in modeling.

Example: The slope of the tangent to the curve $y = f(x)$ is given by $-x/y$. Find an equation of the curve $y = f(x)$.

C.5.3 Solve differential equations of the form $y' = ky$ as applied to growth and decay problems.

Example: The amount of a certain radioactive material was 10 kg a year ago. The amount is now 9 kg. When will it be reduced to 1 kg? Explain your answer.

C.5.4 Use definite integrals to find the area between a curve and the x-axis, or between two curves.

Example: Find the area bounded by $y = \sqrt{x}$, $x = 0$, and $x = 2$.

C.5.5 Use definite integrals to find the average value of a function over a closed interval.

Example: Find the average value of $y = \sqrt{x}$ over $[0, 2]$.

C.5.6 Use definite integrals to find the volume of a solid with known cross-sectional area.

Example: A cone with its vertex at the origin lies symmetrically along the x-axis. The base of the cone is at $x = 5$ and the base radius is 7. Use integration to find the volume of the cone.

C.5.7 Apply integration to model and solve problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.

Example: Find the amount of work done by a variable force.

C

NOTES

Discrete Mathematics

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Discrete Mathematics:

Standard 1 — Counting Techniques

Students develop an understanding of combinatorial reasoning, using various types of diagrams and the fundamental counting principle to find numbers of outcomes and related probabilities. They also use simulations to solve counting and probability problems.

Standard 2 — Matrices

Students understand how matrices can be used to store and organize data and to solve systems of equations. They also use matrices to solve Markov chain problems that link present events to future events using probabilities.

Standard 3 — Recursion

Students understand and apply recursive methods to solve problems, including the use of finite differences.

Standard 4 — Graph Theory

Students understand how graphs of points joined by lines can model a variety of problem situations. These include critical path analysis, graph coloring problems, minimal spanning trees, and bin-packing techniques.

Standard 5 — Social Choice

Students analyze election data to evaluate different election methods and use weighted voting techniques to decide voting power within a group. They understand and use fair division techniques to solve apportionment problems.

Standard 6 — Linear Programming

Students understand how to use diagrams to solve simple optimization problems and extend this to the Simplex method for solving more general optimization problems.

Standard 7 — Game Theory

Students understand and use game theory methods to solve strictly determined games and nonstrictly determined games.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to combinatorial reasoning, recursive thinking, critical path analysis, and other counting situations.

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

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Standard 1

Counting Techniques

Students use counting techniques.

DM.1.1 Use networks, traceable paths, tree diagrams, Venn diagrams, and other pictorial representations to find the number of outcomes in a problem situation.

Example: In a motel there are 4 different elevators that go from Joan's room to the pool and 3 different doors to the pool area. Use a tree diagram to show how many different ways Joan can get from her room to the pool.

DM.1.2 Use the fundamental counting principle to find the number of outcomes in a problem situation.

Example: You are getting dressed one morning when you realize that you have far too many choices. You have 6 shirts to choose from, 4 pairs of jeans, and 3 pairs of shoes. Ignoring color coordination, construct a tree diagram or other pictorial representation to show how many different outfits you could assemble.

DM.1.3 Use combinatorial reasoning to solve problems.

Example: You know that your locker combination contains the numbers 2, 4, 6, and 8, but you have forgotten the order in which they occur. What is the maximum number of combinations you need to try before your locker opens?

DM.1.4 Use counting techniques to solve probability problems.

Example: In the last example, what is the probability that your locker opens with the first combination you try?

DM.1.5 Use simulations to solve counting and probability problems.

Example: A panel of 12 jurors was selected from a large pool that was 70% male and 30% female. The jury turned out to be 11 men and 1 woman. Suspecting gender bias, the defense attorneys asked how likely is it that this situation, or worse, would occur purely by chance. Simulate this situation using a random number generator to select 12 numbers, letting 0, 1, and 2 represent women and 3, 4, 5, 6, 7, 8, and 9 represent men. Note the number of times that 11 or 12 men are chosen.

Standard 2

Matrices

Students use matrices.

DM.2.1 Use matrices to organize and store data.

Example: Central High School offers three different styles of class rings — standard, classic, and deluxe. Each style is available in a girl's ring and a boy's ring. Make up your own data to show how many of each variety was sold and store it in a matrix with rows and columns labeled.

DM.2.2 Use matrix operations to solve problems.

Example: Suppose the rings in the previous problem cost \$90, \$120, and \$135 for the girls' rings and \$110, \$140, and \$165 for the boys' rings. Display this information in a matrix and use matrix multiplication to find the total revenue from the sale of girls' rings and boys' rings.

DM.2.3 Use row-reduction techniques to solve problems.

Example: Solve this system of equations using an augmented matrix and row reduction:

$$\begin{aligned} x - 2y + 3z &= 5 \\ x \quad\quad + 3z &= 11 \\ 5y - 6z &= 9 \end{aligned}$$

DM.2.4 Use the inverse of a matrix to solve problems.

Example: Solve the system of equations in the last example using an inverse matrix.

DM.2.5 Use Markov chains to solve problems.

Example: If a student does homework one day, there is a 70% probability that he or she will do it again the next day. If a student does not do homework one day, there is a 60% probability that he or she will not do it again the next day. On Thursday, 75% of the students did their homework. What can you expect to happen on Friday?

Standard 3

Recursion

Students use recursive techniques.

DM.3.1 Use recursive thinking to solve problems.

Example: How many handshakes would occur in this room if everyone shook hands with everyone else? Create a spreadsheet that will find the number of handshakes starting with one person and increasing the number to 15.

DM.3.2 Use finite differences to solve problems.

Example: Add two columns to the spreadsheet from the previous example and create appropriate formulas for each to calculate first and second differences.

Standard 4

Graph Theory

Students use graph theory techniques.

DM.4.1 Use graphs consisting of vertices and edges to model a problem situation.

Example: There are two islands in the River Seine in Paris. The city wants to construct four bridges that connect each island to each side of the riverbank and one bridge that connects the two islands directly. The city planners want to know if it is possible to start at one point, cross all five bridges, and end up at the same point without crossing a bridge twice. Use a graph to help solve this problem.

DM.4.2 Use critical path analysis to solve scheduling problems.

Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project.

DM.4.3 Use graph coloring techniques to solve problems.

Example: Color a map of the Midwestern states of the United States so that no adjacent states are the same color. What is the minimum number of colors needed?

DM.4.4 Use minimal spanning trees to solve problems.

Example: The telephone company wants to connect cities with new telephone lines. They calculate the cost of connecting each pair of cities, but want to reduce costs by connecting cities through others. Given a graph showing the cost of connecting each pair of cities, find the minimum cost for connecting all the cities with new telephone lines.

DM.4.5 Use bin-packing techniques to solve problems.

Example: Six large crates of electronic equipment are to be shipped to a warehouse. The crates weigh 2,800, 6,000, 5,400, 1,600, 6,800, and 5,000 pounds. Each delivery truck has a capacity of 10,000 pounds. What is the minimum number of trucks needed to send all the crates?

Standard 5

Social Choice

Students use the mathematics of social choice.

DM.5.1 Use election theory techniques to analyze election data.

Example: Each student in your class ranks four kinds of pop from the most preferred to least preferred. Discuss the merits of various methods for deciding on the overall ranking by the class.

DM.5.2 Use weighted voting techniques to decide voting power within a group.

Example: Company stockholders have different numbers of votes according to their holdings. For given holdings, find the power index of each stockholder.

DM.5.3 Use fair division techniques to divide continuous objects.

Example: Find a method for dividing a piece of cake among three people so that each person feels they have received a fair share.

DM.5.4 Use fair division techniques to solve apportionment problems.

Example: Find the enrollment of seniors, juniors, sophomores, and freshmen at your high school. If there are 20 seats on the Student Council, how should the representatives be apportioned so that the voting power of each class is proportional to its size?

Standard 6

Linear Programming

Students use linear programming techniques.

DM.6.1 Use geometric techniques to solve optimization problems.

Example: A company produces two varieties of widgets — standard and deluxe. A standard widget takes 3 hours to assemble and 6 hours to finish. A deluxe widget takes 5 hours to assemble and 5 hours to finish. The assemblers can work no more than 45 hours per week and the finishers can work no more than 60 hours per week. The profit is $32 on a standard widget and $40 on a deluxe widget. Use a graph to find how many of each model should be produced each week to maximize profit.

DM.6.2 Use the Simplex method to solve optimization problems with and without technology.

Example: Use the Simplex method to solve the problem in the last example.

Standard 7

Game Theory

Students use game theory.

DM.7.1 Use game theory to solve strictly determined games.

Example: Consider a card game where John gets a 4 of Hearts and a 5 of Clubs, and Susan gets a 3 of Clubs and a 6 of Hearts. The players each show one card simultaneously. The player who shows the card of larger value receives the sum of the numbers on the two cards shown. Set up the game matrix and find the optimal strategy and the value of the game.

DM.7.2 Use game theory to solve nonstrictly determined games.

Example: In the game "Two-Finger Morra," each of two players shows either one or two fingers. If the total number of fingers shown is even, Player A collects a dollar for each finger shown from Player B. If the total number of fingers is odd, Player A pays $3 to Player B. Set up the game matrix and find the optimal strategy and the value of the game.

NOTES

Geometry

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Geometry:

Standard 1 — Points, Lines, Angles, and Planes

Students understand the relationship between geometric ideas and their representation with coordinate systems. They find lengths and midpoints of line segments, slopes, parallel and perpendicular lines, and equations of lines. They also construct lines and angles, explaining and justifying the processes they use.

Standard 2 — Polygons

Students identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons, etc.), using terms such as regular, convex, and concave. They find measures of angles, sides, perimeters, and areas of polygons, justifying their methods. They apply transformations to polygons and they relate geometry to algebra by using coordinate geometry to determine congruence, similarity, symmetry, and tessellations.

Standard 3 — Quadrilaterals

Students classify and understand relationships among quadrilaterals (rectangle, parallelogram, kite, etc.). They find measures of sides, perimeters, and areas of quadrilaterals, justifying their methods. They relate geometry to algebra by using coordinate geometry to determine regularity, congruence, and similarity. They use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas.

Standard 4 — Triangles

Students identify and describe various kinds of triangles (right, acute, scalene, isosceles, etc.). They define and construct altitudes, medians, and bisectors, and triangles congruent to given triangles. They prove that triangles are congruent or similar and use properties of these triangles to solve problems involving lengths and areas. They find measures of sides, perimeters, and areas of triangles, justifying their methods. They relate geometry to algebra by using coordinate geometry to determine regularity, congruence, and similarity. They understand and apply the inequality theorems of triangles.

Standard 5 — Right Triangles

Students prove the Pythagorean Theorem and apply it to solving problems, including those involving the altitudes of right triangles and triangles with special angle relationships. They define and understand the concepts of the trigonometric functions (sine, tangent, etc.), know and use basic relationships among these functions, and apply their knowledge of trig functions to solving word problems.

Standard 6 — Circles

Students define and understand ideas related to circles (radius, tangent, chord, etc.). They perform constructions and prove theorems related to circles. They find measures of arcs and angles related to them, as well as measures of circumference and area, and they relate geometry to algebra by finding the equation of a circle in the coordinate plane.

Standard 7 — Polyhedra and Other Solids

Students describe and make regular and nonregular polyhedra (cube, pyramid, tetrahedron, octahedron, etc.). They explore relationships among the faces, edges, and vertices of polyhedra. They describe sets of points on spheres, using terms such as great circle. They describe symmetries of solids and understand the properties of congruent and similar solids.

Standard 8 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to making conjectures, using axioms and theorems, understanding the converse and contrapositive of a statement, constructing logical arguments, and writing geometric proofs. They also learn about inductive and deductive reasoning and how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

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Standard 1

Points, Lines, Angles, and Planes

Students find lengths and midpoints of line segements. They describe and use parallel and perpendicular lines. They find slopes and equations of lines.

G.1.1 **Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems.**

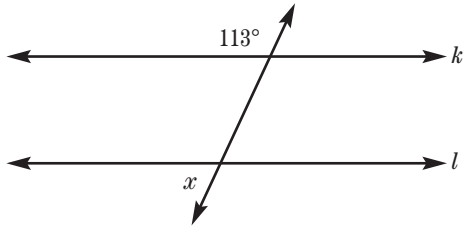
Example: Find the length and midpoint of the line joining the points $A\ (3, 8)$ and $B\ (9, 0)$.

G.1.2 **Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass, explaining and justifying the process used.**

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

G.1.3 **Understand and use the relationships between special pairs of angles formed by parallel lines and transversals.**

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.



G.1.4 **Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.**

Example: Find an equation of a line perpendicular to $y = 4x - 2$.

Standard 2

Polygons

Students identify and describe polygons and measure interior and exterior angles. They use congruence, similarity, symmetry, tessellations, and transformations. They find measures of sides, perimeters, and areas.

G.2.1 Identify and describe convex, concave, and regular polygons.

Example: Draw a regular hexagon. Is it convex or concave? Explain your answer.

G.2.2 Find measures of interior and exterior angles of polygons, justifying the method used.

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

G.2.3 Use properties of congruent and similar polygons to solve problems.

Example: Divide a regular hexagon into triangles by joining the center to each vertex. Show that these triangles are all the same size and shape and find the sizes of the interior angles of the hexagon.

G.2.4 Apply transformations (slides, flips, turns, expansions, and contractions) to polygons to determine congruence, similarity, symmetry, and tessellations. Know that images formed by slides, flips, and turns are congruent to the original shape.

Example: Use a drawing program to create regular hexagons, regular octagons, and regular pentagons. Under the drawings, describe which of the polygons would be best for tiling a rectangular floor. Explain your reasoning.

G.2.5 Find and use measures of sides, perimeters, and areas of polygons. Relate these measures to each other using formulas.

Example: A rectangle of area 360 square yards is ten times as long as it is wide. Find its length and width.

G.2.6 Use coordinate geometry to prove properties of polygons such as regularity, congruence, and similarity.

Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6), and (1, 5) a square?

Standard 3

Quadrilaterals

Students identify and describe simple quadrilaterals. They use congruence and similarity. They find measures of sides, perimeters, and areas.

G.3.1 **Describe, classify, and understand relationships among the quadrilaterals square, rectangle, rhombus, parallelogram, trapezoid, and kite.**

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid, and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

G.3.2 **Use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas.**

Example: Of two similar rectangles, the second has sides three times the length of the first. How many times larger in area is the second rectangle?

G.3.3 **Find and use measures of sides, perimeters, and areas of quadrilaterals. Relate these measures to each other using formulas.**

Example: A section of roof is a trapezoid with length 4 m at the ridge and 6 m at the gutter. The shortest distance from ridge to gutter is 3 m. Construct a model using a drawing program, showing how to find the area of this section of roof.

G.3.4 **Use coordinate geometry to prove properties of quadrilaterals, such as regularity, congruence, and similarity.**

Example: Is rectangle *ABCD* with vertices at (0, 0), (4, 0), (4, 2), (0, 2) congruent to rectangle *PQRS* with vertices at (-2, -1), (2, -1), (2, 1), (-2, 1)?

Standard 4

Triangles

Students identify and describe types of triangles. They identify and draw altitudes, medians, and angle bisectors. They use congruence and similarity. They find measures of sides, perimeters, and areas. They apply inequality theorems.

G.4.1 **Identify and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular.**

Example: Use a drawing program to create examples of right, acute, obtuse, scalene, isosceles, equilateral, and equiangular triangles. Identify and describe the attributes of each triangle.

G.4.2 **Define, identify, and construct altitudes, medians, angle bisectors, and perpendicular bisectors.**

Example: Draw several triangles. Construct their angle bisectors. What do you notice?

G.4.3 **Construct triangles congruent to given triangles.**

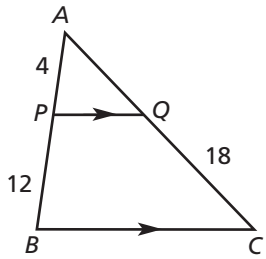
Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

G.4.4 **Use properties of congruent and similar triangles to solve problems involving lengths and areas.**

Example: Of two similar triangles, the second has sides half the length of the first. The area of the first triangle is 20 cm^2. What is the area of the second?

G.4.5 **Prove and apply theorems involving segments divided proportionally.**

Example: In triangle ABC, $\overline{PQ}$ is parallel to $\overline{BC}$. What is the length of $\overline{AQ}$?



G.4.6 **Prove that triangles are congruent or similar and use the concept of corresponding parts of congruent triangles.**

Example: In the last example, prove that triangles ABC and APQ are similar.

G.4.7 **Find and use measures of sides, perimeters, and areas of triangles. Relate these measures to each other using formulas.**

Example: The gable end of a house is a triangle 20 feet long and 13 feet high. Find its area.

G.4.8 **Prove, understand, and apply the inequality theorems: triangle inequality, inequality in one triangle, and the hinge theorem.**

Example: Can you draw a triangle with sides of length 7 cm, 4 cm, and 15 cm?

G.4.9 **Use coordinate geometry to prove properties of triangles such as regularity, congruence, and similarity.**

Example: Draw a triangle with vertices at (1, 3), (2, 5), and (6, 1). Draw another triangle with vertices at (-3, -1), (-2, 1), and (2, -3). Are these triangles the same shape and size?

Standard 5

Right Triangles

Students prove the Pythagorean Theorem and use it to solve problems. They define and apply the trigonometric relations sine, cosine, and tangent.

G.5.1 **Prove and use the Pythagorean Theorem.**

Example: On each side of a right triangle, draw a square with that side of the triangle as one side of the square. Find the areas of the three squares. What relationship is there between the areas?

G.5.2 **State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.**

Example: In triangle ABC with right angle at C, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

G.5.3 **Use special right triangles (30° - 60° and 45° - 45°) to solve problems.**

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

G.5.4 **Define and use the trigonometric functions (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles.**

Example: In triangle ABC, $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cot A$.

G.5.5 **Know and use the relationship $\sin^2 x + \cos^2 x = 1$.**

Example: Show that, in a right triangle, $\sin^2 x + \cos^2 x = 1$ is an example of the Pythagorean Theorem.

G.5.6 **Solve word problems involving right triangles.**

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

Standard 6

Circles

Students define ideas related to circles: e.g., radius, tangent. They find measures of angles, lengths, and areas. They prove theorems about circles. They find equations of circles.

G.6.1 Find the center of a given circle. Construct the circle that passes through three given points not on a line.

Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

G.6.2 Define and identify relationships among: radius, diameter, arc, measure of an arc, chord, secant, and tangent.

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

G.6.3 Prove theorems related to circles.

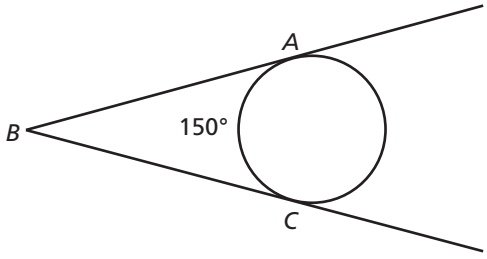
Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

G.6.4 Construct tangents to circles and circumscribe and inscribe circles.

Example: Draw an acute triangle and construct the circumscribed circle.

G.6.5 Define, find, and use measures of arcs and related angles (central, inscribed, and intersections of secants and tangents).

Example: Find the measure of angle *ABC* in the diagram below.



G.6.6 Define and identify congruent and concentric circles.

Example: Are circles with the same center always the same shape? Are they always the same size?

G.6.7 Define, find, and use measures of circumference, arc length, and areas of circles and sectors. Use these measures to solve problems.

Example: Which will give you more: three 6-inch pizzas or two 8-inch pizzas? Explain your answer.

G.6.8 Find the equation of a circle in the coordinate plane in terms of its center and radius.

Example: Find the equation of the circle with radius 10 and center (6, -3).

Standard 7

Polyhedra and Other Solids

Students describe and make polyhedra and other solids. They describe relationships and symmetries, and use congruence and similarity.

G.7.1 Describe and make regular and nonregular polyhedra.

Example: Is a cube a regular polyhedron? Explain why or why not.

G.7.2 Describe the polyhedron that can be made from a given net (or pattern). Describe the net for a given polyhedron.

Example: Make a net for a tetrahedron out of poster board and fold it up to make the tetrahedron.

G.7.3 Describe relationships between the faces, edges, and vertices of polyhedra.

Example: Count the sides, edges, and corners of a square-based pyramid. How are these numbers related?

G.7.4 Describe symmetries of geometric solids.

Example: Describe the rotation and reflection symmetries of a square-based pyramid.

G.7.5 Describe sets of points on spheres: chords, tangents, and great circles.

Example: On Earth, is the equator a great circle?

G.7.6 Identify and know properties of congruent and similar solids.

Example: Explain how the surface area and volume of similar cylinders are related.

G.7.7 Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.

Example: An ice cube is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the ice cube?

Standard 8

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

G.8.1 Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: How far does the tip of the minute hand of a clock move in 20 minutes if the tip is 4 inches from the center of the clock?

G.8.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 12 inches. Is his answer reasonable? Why or why not?

Students develop and evaluate mathematical arguments and proofs.

G.8.3 Make conjectures about geometric ideas. Distinguish between information that supports a conjecture and the proof of a conjecture.

Example: Calculate the ratios of side lengths in several different-sized triangles with angles of 90°, 50°, and 40°. What do you notice about the ratios? How might you prove that your observation is true (or show that it is false)?

G.8.4 Write and interpret statements of the form "if – then" and "if and only if."

Example: Decide whether this statement is true: "If today is Sunday, then we have school tomorrow."

G.8.5 State, use, and examine the validity of the converse, inverse, and contrapositive of "if – then" statements.

Example: In the last example, write the converse of the statement.

G.8.6 Identify and give examples of undefined terms, axioms, and theorems, and inductive and deductive proofs.

Example: Do you prove axioms from theorems or theorems from axioms?

G.8.7 Construct logical arguments, judge their validity, and give counterexamples to disprove statements.

Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent.

G.8.8 Write geometric proofs, including proofs by contradiction and proofs involving coordinate geometry. Use and compare a variety of ways to present deductive proofs, such as flow charts, paragraphs, and two-column and indirect proofs.

Example: In triangle LMN, $LM = LN$. Prove that $\angle LMN \cong \angle LNM$.

G.8.9 Perform basic constructions, describing and justifying the procedures used. Distinguish between constructing and drawing geometric figures.

Example: Construct a line parallel to a given line through a given point not on the line, explaining and justifying each step.

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Integrated Mathematics I:

Standard 1 — Number Sense and Computation

Students deepen their understanding of real numbers by comparing expressions involving square roots and exponents. They use the properties of real numbers to simplify algebraic formulas and they use dimensional analysis to convert between different measurement units.

Standard 2 — Algebra and Functions

Students solve and graph linear equations and inequalities using order properties of the real numbers and they solve word problems involving linear equations, inequalities, and formulas. Students solve pairs of linear equations in two variables using both graphs and algebraic methods. Students operate with polynomials — adding, subtracting, multiplying, dividing, and raising to powers. They draw graphs of quadratic, cubic, and rational functions and solve problems of growth and decay.

Standard 3 — Geometry and Measurement

Students identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons, etc.), using terms such as regular, convex, and concave. They find measures of sides, perimeters, and areas of polygons, justifying their methods, and they apply transformations to polygons. Students prove the Pythagorean Theorem and apply it to solving problems. They explore relationships among the faces, edges, and vertices of polyhedra and describe symmetries of solids.

Standard 4 — Data Analysis and Statistics

Students will use and analyze a variety of data displays: line plots, histograms, stem-and-leaf plots, frequency tables, and scatterplots. They will understand a number of measures of central tendency and variability, as well as recognize patterns in tables and graphs of linear data.

Standard 5 — Probability

Students will use simulations, find empirical and theoretical probabilities, and use the Law of Large Numbers. They will understand independent events and probability distributions.

Standard 6 — Discrete Mathematics

Students will construct vertex-edge graphs and digraphs and will use Euler paths and recursion equations to solve problems. They will use matrices to describe vertex-edge graphs and they will find row and column sums for matrices.

Standard 7 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn about inductive and deductive reasoning and how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Number Sense and Computation

Students simplify and compare expressions. They use rational exponents and simplify square roots.

IM1.1.1 Compare real number expressions.

Example: Which is larger: 2^3 or $\sqrt{49}$?

IM1.1.2 Simplify square roots using factors.

Example: Explain why $\sqrt{48} = 4\sqrt{3}$.

IM1.1.3 Understand and use the distributive, associative, and commutative properties.

Example: Simplify $(6x^2 - 5x + 1) - 2(x^2 + 3x - 4)$ by removing the parentheses and rearranging. Explain why you can carry out each step.

IM1.1.4 Use the laws of exponents for rational exponents.

Example: Simplify $25^{3/2}$.

IM1.1.5 Use dimensional (unit) analysis to organize conversions and computations.

Example: Convert 5 miles per hour to feet per second: $\frac{5 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \approx 7.3 \text{ ft/sec}$.

Standard 2

Algebra and Functions

Students solve linear equations and inequalities in one variable. They write equations of lines and find and use the slope and y-intercept of lines. Students solve pairs of linear equations using graphs and algebra. Students add, subtract, multiply, and divide polynomials and solve word problems using exponential functions.

IM1.2.1 Solve linear equations.

Example: Solve the equation $7a + 2 = 5a - 3a + 8$.

IM1.2.2 Solve equations and formulas for a specified variable.

Example: Solve the equation $q = 4p - 11$ for p.

IM1.2.3 Find solution sets of linear inequalities when possible numbers are given for the variable.

Example: Solve the inequality $6x - 3 > 10$ for x in the set $\{0, 1, 2, 3, 4\}$.

IM1.2.4 Solve linear inequalities using properties of order.

Example: Solve the inequality $8x - 7 \leq 2x + 5$, explaining each step in your solution.

IM1.2.5 Solve word problems that involve linear equations, formulas, and inequalities.

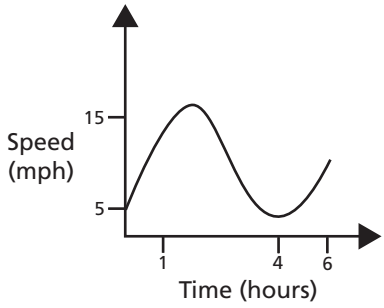
Example: You are selling tickets for a play that cost \$3 each. You want to sell at least \$50 worth. Write and solve an inequality for the number of tickets you must sell.

IM1.2.6 Sketch a reasonable graph for a given relationship.

Example: Sketch a reasonable graph for a person's height from age 0 to 25.

IM1.2.7 Interpret a graph representing a given situation.

Example: Jessica is riding a bicycle. The graph below shows her speed as it relates to the time she has spent riding. Describe what might have happened to account for such a graph.



IM1.2.8 Understand the concept of a function, decide if a given relation is a function, and link equations to functions.

Example: Use either paper or a spreadsheet to generate a list of values for x and y in $y = x^2$. Based on your data, make a conjecture about whether or not this relation is a function. Explain your reasoning.

IM1.2.9 Find the domain and range of a relation.

Example: Based on the list of values from the example in indicator 2.8, what is the domain and range of $y = x^2$?

IM1.2.10 Graph a linear equation.

Example: Graph the equation $3x - y = 2$.

IM1.2.11 Find the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

Example: Find the slope and y-intercept of the line $4x + 6y = 12$.

IM1.2.12 Write the equation of a line in slope-intercept form. Understand how the slope and y-intercept of the graph are related to the equation.

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

IM1.2.13 Write the equation of a line given appropriate information.

Example: Find an equation of the line through the points $(1, 4)$ and $(3, 10)$, then find an equation of the line through the point $(1, 4)$ perpendicular to the first line.

IM1.2.14 Write the equation of a line that models a data set and use the equation (or the graph of the equation) to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

Example: As your family is traveling along an interstate, you note the distance traveled every 5 minutes. A graph of time and distance shows that the relation is approximately linear. Write the equation of the line that fits your data. Predict the time for a journey of 50 miles. What does the slope represent?

IM1.2.15 Use a graph to estimate the solution of a pair of linear equations in two variables.

Example: Graph the equations $3y - x = 0$ and $2x + 4y = 15$ to find where the lines intersect.

IM1.2.16 Understand and use the substitution method to solve a pair of linear equations in two variables.

Example: Solve the equations $y = 2x$ and $2x + 3y = 12$ by substitution.

IM1.2.17 Understand and use the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use subtraction to solve the equations: $3x + 4y = 11$ and $3x + 2y = 7$.

IM1.2.18 Understand and use multiplication with the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use multiplication with the subtraction method to solve the equations: $x + 4y = 16$ and $3x + 2y = -3$.

IM1.2.19 Use pairs of linear equations to solve word problems.

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$ and the expenses for that product can be represented by the equation $y = 5.25x + 10{,}000$, where x is the amount of the product sold and y is the number of dollars. How much of the product must be sold for the company to reach the break-even point?

IM1.2.20 Add and subtract polynomials.

Example: Simplify $(4x^2 - 7x + 2) - (x^2 + 4x - 5)$.

IM1.2.21 Multiply and divide monomials.

Example: Simplify $a^2b^5 \div ab^2$.

IM1.2.22 Find powers and roots of monomials (only when the answer has an integer exponent).

Example: Find the square root of a^2b^6.

IM1.2.23 Multiply polynomials.

Example: Multiply $(n + 2)(4n - 5)$.

IM1.2.24 Divide polynomials by monomials.

Example: Divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.

IM1.2.25 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x-intercepts of $y = 3x^2 - 5x - 1$ as the solutions of the equation.

IM1.2.26 Graph quadratic, cubic, and radical equations.

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet (generate a data set), display the graph to check your work.

IM1.2.27 Solve quadratic equations using the quadratic formula.

Example: Solve the equation $x^2 - 7x + 9 = 0$.

IM1.2.28 Use quadratic equations to solve word problems.

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

IM1.2.29 Use graphing technology to find approximate solutions of quadratic and cubic equations.

Example: Use a graphing calculator to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth.

IM1.2.30 Graph exponential functions.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

IM1.2.31 Solve word problems involving applications of exponential functions to growth and decay.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

Standard 3

Geometry and Measurement

Students identify and describe polygons, including finding measures of sides, perimeters, and areas. They use congruence, similarity, symmetry, tessellations, and transformations. Students understand the Pythagorean Theorem and use it to solve problems. They describe relationships and symmetries in geometric solids.

IM1.3.1 Identify and describe convex, concave, and regular polygons.

Example: Draw a regular hexagon. Is it convex or concave? Explain your answer.

IM1.3.2 Apply transformations (slides, flips, turns, expansions, and contractions) to polygons to determine congruence, similarity, symmetry, and tessellations. Know that images formed by slides, flips, and turns are congruent to the original shape.

Example: Use a drawing program to create regular hexagons, regular octagons, and regular pentagons. Under the drawings, describe which of the polygons would be best for tiling a rectangular floor. Explain your reasoning.

IM1.3.3 Find and use measures of sides, perimeters, and areas of polygons. Relate these measures to each other using formulas.

Example: A rectangle of area 360 square yards is 10 times as long as it is wide. Find its length and width.

IM1.3.4 Use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas.

Example: Of two similar rectangles, the second has sides three times the length of the first. How many times larger in area is the second rectangle?

IM1.3.5 Find and use measures of sides, perimeters, and areas of quadrilaterals. Relate these measures to each other using formulas.

Example: A section of roof is a trapezoid with length 4 m at the ridge and 6 m at the gutter. The shortest distance from ridge to gutter is 3 m. Construct a model using a drawing program, showing how to find the area of this section of roof.

IM1.3.6 Prove and use the Pythagorean Theorem.

Example: On each side of a right triangle, draw a square with that side of the triangle as one side of the square. Find the areas of the three squares. What relationship is there between the areas?

IM1.3.7 Describe relationships between the faces, edges, and vertices of polyhedra.

Example: Count the sides, edges, and corners of a square-based pyramid. How are these numbers related?

IM1.3.8 Describe symmetries of geometric solids.

Example: Describe the rotation and reflection symmetries of a square-based pyramid.

Standard 4

Data Analysis and Statistics

Students find measures of the center and variability of a set of data, as well as construct and analyze data displays and plot least square regression lines.

IM1.4.1 Construct a line plot.

Example: The number of people living on American farms has been decreasing. Construct a line plot for the data, given the number of Americans (in millions) living on farms: (1940, 30.5) (1950, 23.0) (1960, 15.6) (1970, 9.7) (1980, 7.2)

IM1.4.2 Find measures of central tendency for a set of data.

Example: The following gives the number of home runs hit by Roger Maris in his 10 years as a New York Yankee: 13, 23, 26, 16, 33, 61, 28, 39, 14, 8. Determine the mean, median, and mode of this data.

IM1.4.3 Find skewness and symmetry from a graph of data.

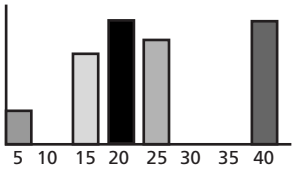
Example: Discuss the skewness of this graph.

IM1.4.4 Construct a histogram using a graphing calculator.

Example: The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Make a histogram using a graphing calculator.

IM1.4.5 Identify clusters, gaps, and outliers for a set of data.

Example: Describe gaps for the shown data.



IM1.4.6 Find a linear transformation.

Example: Consider the following data: 6, 4, 4, 6, 8, 10, 2, 5, 9. Suppose that 5 is added to each value. Compare the mean and average mean deviation of the original and new data sets.

IM1.4.7 Construct a stem-and-leaf plot using a graphing calculator.

Example: The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Construct a stem-and-leaf plot.

IM1.4.8 Find the mean absolute deviation for a set of data.

Example: The following gives the number of keys carried by six students: 3, 3, 6, 0, 4, 4. Calculate the mean, then determine the absolute deviation of each value and the mean absolute deviation.

IM1.4.9 Find the standard deviation and describe its properties.

Example: The following gives the number of keys carried by six students: 3, 3, 6, 0, 4, 4. Calculate the mean. Next, determine the square of the difference between the mean and each value and then determine the standard deviation.

IM1.4.10 Construct a frequency table for a set of data.

Example: The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Make a frequency table with a first class (interval) of 55-59.

IM1.4.11 Summarize and interpret sets of data using center and variability.

Example: The following gives the pulse rates of 20 students: 55, 95, 62, 94, 93, 91, 64, 67, 80, 80, 82, 70, 72, 76, 88, 84, 88, 86, 78, 78. Find the mean, range, quartiles, and interquartile range.

IM1.4.12 Construct a scatterplot from a set of data.

Example: The following gives data concerning (age in months, height in centimeters) of a child: (36, 86) (48, 90) (51, 91) (54, 93) (57, 94) (60, 95). Construct a scatterplot for this data.

IM1.4.13 Calculate the sum of squared differences for a set of data.

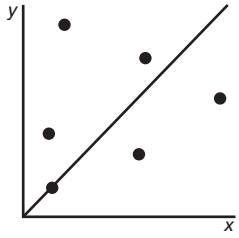
Example: The following gives data concerning (age in months, height in centimeters) of a child: (36, 86) (48, 90) (51, 91) (54, 93) (57, 94) (60, 95). The least squares regression line for these data is $h = 72.0 + .4a$. For each data point, calculate the square of the difference between the observed and predicted values. Calculate the sum of these values.

IM1.4.14 Plot the least square regression line from a set of data.

Example: The following gives data concerning (age in months, height in centimeters) of a child: (36, 86) (48, 90) (51, 91) (54, 93) (57, 94) (60, 95). Use a graphing calculator to determine the equation of the least squares regression line.

IM1.4.15 Compare sets of data using scatterplots and the line $y = x$, and interpret these comparisons for real-world data.

Example: Six students were ranked on an English test (x) and a mathematics test (y). The results along with the line $y = x$ are shown on the scatterplot. How many students scored higher in mathematics than English?



IM1.4.16 Recognize patterns in tables and graphs that are modeled by linear equations.

Example: Write a "$y =$ equation" for the following table of values.

X	Y
0	50
1	46
2	42
3	38

Standard 5

Probability

Students use simulations, find probabilities, and use the Law of Large Numbers.

IM1.5.1 Design and use simulations in order to estimate answers related to probability.

Example: A student is taking a true/false test. He is rolling a die to choose an answer. Assume that true and false are both equally likely to be correct answers. Explain how to use a single die to choose a true or false answer.

IM1.5.2 Use empirical (experimental) and theoretical probabilities.

Example: A student flipped two coins. The result was two tails. Find the theoretical probability of flipping two coins and getting two tails. Explain your answer.

IM1.5.3 Understand independent events.

Example: The experiment is rolling a single die. Event A is rolling an even number. Event B is rolling a number greater than three. Are event A and event B independent events? Give an event C that is independent of event B.

IM1.5.4 Use the Law of Large Numbers to understand situations involving chance.

Example: A class is flipping coins to study probability. In one experiment, the coin was flipped 1,000 times. The next day the coin was flipped 2,000 times. Which experiment — the 1,000 flips or the 2,000 flips — has the highest probability of getting 50 percent heads? Why?

IM1.5.5 Understand the concept of a probability distribution. Understand how an approximate probability can be constructed using simulation involving chance.

Example: A class of 25 students is conducting a probability experiment. All the students are standing. Each student flips a coin. If the result is heads, the student sits down. How many students are expected to be standing after the first toss? How many flips are expected until all of the students are seated?

Standard 6

Discrete Mathematics

Students construct graphs, explore algorithms, and use recursion equations and matrices.

IM1.6.1 Construct vertex-edge graph models involving relationships among a finite number of elements.

Example: Bedford High School has the following committee membership.

Executive — Lehman, Smith, Rupp, George
Academic — Smith, Rupp, Vorndran
Extracurriculars — Yoder, Spring
Social — Yoder, Rupp, Jackson
Homecoming — Spring, Marshall, Simpson

Make a vertex-edge graph. Represent the committees with vertices. If two committees share a person, then connect the vertices with an edge. What is the minimum number of meeting times needed so everyone can attend his or her committees' meetings?

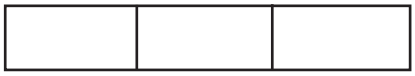
IM1.6.2 Construct digraphs.

Example: An athletic conference with four teams has just completed competition. The following matrix shows which team won in head-to-head competition. The "1" in row 1 column 2 means team A won against team B. Use the matrix information to make a digraph showing the same information.

	A	B	C	D
A	0	1	1	0
B	0	0	1	1
C	0	0	0	0
D	1	0	1	0

IM1.6.3 Use Euler paths and circuits to solve real-world problems.

Example: The diagram shows a three-block area in a city. A mail carrier must deliver mail in this three-block area. Find a delivery route that is an Euler circuit or modify the diagram to make an Euler circuit possible.



IM1.6.4 **Develop the skill of algorithmic problem solving: designing, using, and analyzing systematic procedures for problem solving.**

Example: The formula $B = (1/16)(D^2 - 8D + 16)$ describes the number of board feet (B) in a log with a diameter in D in inches and length L in feet. Find the number of board feet in a log with a diameter of 8 inches and having a length of 6 feet.

IM1.6.5 **Use a recursion function to describe an exponential function.**

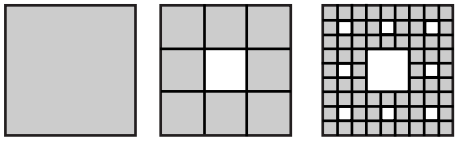
Example: You have just cut your finger with a knife and 10 bacteria cells have entered the wound. This strain of bacteria doubles every 30 minutes. Model the growth of this bacteria using a recursive function. Provide a table of values for the first five hours of the growth of the bacteria.

IM1.6.6 **Use a variety of recursion equations to describe a function.**

Example: You have saved $1,000 from summer work and have placed it in a savings account at the local bank. This account earns 5 percent compounded annually. You plan to add $1,000 at the end of each year. Model this investment with a recursive function.

IM1.6.7 **Use a recursion function to describe a fractal.**

Example: Sierpinski's carpet is a fractal pattern. Starting with a solid square "carpet" one meter on a side, smaller and smaller squares are removed from the carpet. The first two stages in forming carpet are shown below. Find recursive and function formulas for the sequence of carpet are that remains at each stage.



Stage 0 Stage 1 Stage 2

IM1.6.8 **Use an adjacency matrix to describe a vertex-edge graph.**

Example: Use an adjacency matrix to represent the vertex-edge graph from indicator 6.1.

IM1.6.9 **Perform row and column sums for matrix equations.**

Example: The Acme construction firm builds one-story and two-story houses. The expenses for fees, labor, and materials are shown in the following matrix. Determine the row and column sums of the matrix and explain the meaning of each sum.

	Fees	Materials	Labor
One-Story House	$2,000	$75,000	$25,000
Two-Story House	$2,500	$100,000	$35,000

Standard 7

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems and develop and evaluate mathematical arguments and proofs.

IM1.7.1 Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: Fran has scored 16, 23, and 30 points in her last three games. How many points must she score in the next game so that her four-game average does not fall below 20 points?

IM1.7.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the example from indicator 7.1 is 10 points. Is his answer reasonable? Why or why not?

IM1.7.3 Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Given an argument (such as $3x + 7 > 5x + 1$, and therefore $-2x > -6$, and therefore $x > 3$), provide a visual presentation of a step-by-step check, highlighting any errors in the argument.

IM1.7.4 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: Try "solving" the equations $x + 3y = 5$ and $5x + 15y = 25$ simultaneously. Explain what went wrong.

IM1.7.5 Decide whether a given algebraic statement is true always, sometimes, or never (statements involving linear or quadratic expressions, equations, and inequalities).

Example: Is the statement $x^2 - 5x + 2 = x^2 + 5x + 2$ true for all x, for some x, or for no x? Explain your answer.

IM1.7.6 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

Example: What type of reasoning are you using when you look for a pattern?

IM1.7.7 Use counterexamples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

Example: Use the demonstration-graphing calculator on an overhead projector to produce an example showing that this statement is false: all quadratic equations have two different solutions.

IM1

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Integrated Mathematics II:

Standard 1 — Algebra and Functions

Students draw and analyze graphs of linear inequalities in two variables and quadratics. They model data with linear equations and make predictions from the results.

Standard 2 — Geometry and Measurement

Students find lengths and midpoints of line segments, slopes, parallel and perpendicular lines, and equations of lines. They also construct lines and angles, explaining and justifying the processes they use. They define and construct altitudes, medians, bisectors, and triangles congruent to given triangles, as well as prove that triangles are congruent or similar and use properties of these triangles to solve problems involving lengths and areas. They find measures of sides, perimeters, and areas of triangles, justifying their methods. They define and understand the concepts of the trigonometric functions and know and use basic relationships among these functions. Students define and understand ideas related to circles (radius, tangent, chord, etc.).

Standard 3 — Data Analysis and Statistics

Students will interpret scatterplots and correlation coefficients and make predictions based on the least squares line.

Standard 4 — Probability

Students will construct probability distributions by simulation and explore the geometric distribution. They will use fundamental concepts of probability and counting principles to solve problems.

Standard 5 — Discrete Mathematics

Students will use graphs and networks to solve problems. They use matrices to organize data and solve problems.

Standard 6 — Trigonometry

Students will explore properties and applications of trigonometric ratios for right triangles.

Standard 7 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn how to use counterexamples to show that a general statement is false. Students apply these skills to making conjectures, using axioms and theorems, understanding the converse and contrapositive of a statement, constructing logical arguments, and writing geometric proofs. They also learn how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

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Standard 1

Algebra and Functions

Students graph linear inequalities in two variables and quadratics. They model data with linear equations.

IM2.1.1 Graph a linear inequality in two variables.

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

IM2.1.2 Interpret given situations as functions in graphs, formulas, and words.

Example: You and your parents are going to Boston and want to rent a car at Logan International Airport on a Monday morning and drop the car off in downtown Providence, R.I., on the following Wednesday. Find the rates from two national car companies and plot the costs on a graph. Decide which company offers the best deal. Explain your answer.

IM2.1.3 Find a linear equation that models a data set using the median fit method and use the model to make predictions.

Example: You light a candle and record its height in centimeters every minute. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.5), (3, 14.8), (4, 13.2), (5, 11.5), (6, 10.0), (7, 8.2), (9, 4.9), and (10, 3.1). Find the median fit line to express the candle's height as a function of the time and state the meaning of the slope in terms of the burning candle.

IM2.1.4 Graph quadratic functions. Apply transformations to quadratic functions. Find and interpret the zeros and maximum or minimum value of quadratic functions.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

Standard 2

Geometry and Measurement

Students identify and describe types of triangles. They define and apply the trigonometric relations. Students apply theorems to triangles and circles.

IM2.2.1 Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems.

Example: Find the length and midpoint of the line joining the points A (3, 8) and B (9, 0).

IM2.2.2 Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass, explaining and justifying the process used.

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

IM2.2.3 Find measures of interior and exterior angles of polygons, justifying the method used.

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

IM2

IM2.2.4 Identify and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular.

Example: Use a drawing program to create examples of right, acute, obtuse, scalene, isosceles, equilateral, and equiangular triangles. Identify and describe the attributes of each triangle.

IM2.2.5 Define, identify, and construct altitudes, medians, angle bisectors, and perpendicular bisectors.

Example: Draw several triangles. Construct their angle bisectors. What do you notice?

IM2.2.6 Use properties of congruent and similar triangles to solve problems involving lengths and areas.

Example: Of two similar triangles, the second has sides half the length of the first. The area of the first triangle is 20 cm^2. What is the area of the second?

IM2.2.7 Find and use measures of sides, perimeters, and areas of triangles. Relate these measures to each other using formulas.

Example: The gable end of a house is a triangle 20 feet long and 13 feet high. Find its area.

IM2.2.8 Prove, understand, and apply the inequality theorems: triangle inequality, inequality in one triangle, and the hinge theorem.

Example: Can you draw a triangle with sides of length 7 cm, 4 cm, and 15 cm?

IM2.2.9 State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.

Example: In triangle ABC with right angle at C, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

IM2.2.10 Use special right triangles (30° - 60° and 45° - 45°) to solve problems.

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

IM2.2.11 Define and use the trigonometric functions (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles.

Example: In triangle ABC, $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cot A$.

IM2.2.12 Know and use the relationship $\sin^2 x + \cos^2 x = 1$.

Example: Show that, in a right triangle, $\sin^2 x + \cos^2 x = 1$ is an example of the Pythagorean Theorem.

IM2.2.13 Solve word problems involving right triangles.

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

IM2.2.14 Find the center of a given circle. Construct the circle that passes through three given points not on a line.

Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

IM2.2.15 Define and identify relationships among: radius, diameter, arc, measure of an arc, chord, secant, and tangent.

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

IM2.2.16 Prove theorems related to circles.

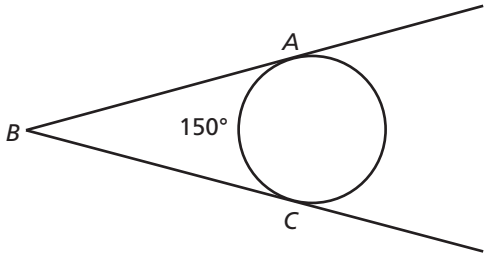
Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

IM2.2.17 Construct tangents to circles and circumscribe and inscribe circles.

Example: Draw an acute triangle and construct the circumscribed circle.

IM2.2.18 Define, find, and use measures of arcs and related angles (central, inscribed, and intersections of secants and tangents).

Example: Find the measure of angle *ABC* in the diagram below.



IM2.2.19 Define and identify congruent and concentric circles.

Example: Are circles with the same center always the same shape? Are they always the same size?

IM2.2.20 Define, find, and use measures of circumference, arc length, and areas of circles and sectors. Use these measures to solve problems.

Example: Which will give you more: three 6-inch pizzas or two 8-inch pizzas? Explain your answer.

IM2.2.21 Describe sets of points on spheres: chords, tangents, and great circles.

Example: On Earth, is the equator a great circle?

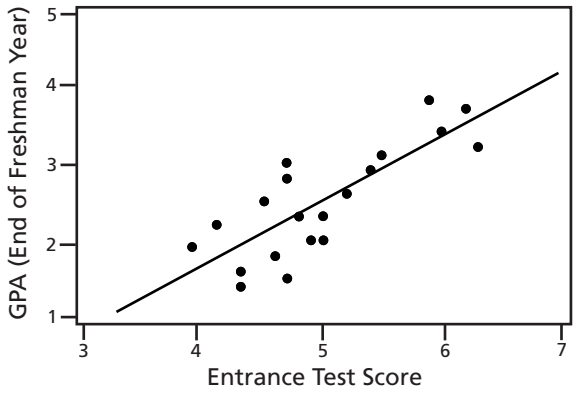
Standard 3

Data Analysis and Statistics

Students interpret scatterplots and analyze correlation.

IM2.3.1 Describe the association between two variables by interpreting a scatterplot.

Example:



The director of admissions of a small college administered a newly designed entrance test to 20 students selected at random from the new freshman class in a study to determine whether a student's grade point average at the end of the freshman year (y) can be predicted from the entrance test score (x). The scatterplot shows a positive relationship between entrance test score and GPA at the end of the freshman year. As the entrance test score increases, the student's GPA at the end of the freshman year also increases.

IM2.3.2 Interpret correlation coefficients.

Example: In the example in indicator 3.1, the correlation coefficient for the student's GPA at the end of the freshman year and the entrance test score is .809, resulting in a strong linear relationship between the two variables.

IM2.3.3 Make predictions from the least squares regression line or its equation.

Example: In the example in indicator 3.1, based on the regression line and the regression equation $y = -1.70 + 0.840x$, a student who scores a 6 on the entrance test score could expect to receive a GPA score of 3.34 at the end of the freshman year.

IM2.3.4 Understand that a correlation between two variables does not necessarily imply one directly causes the other.

Example: Comparing weekly flu medication sales and weekly sweater sales for an area with extreme seasons would exhibit a positive association given that sales for both tend to increase during the winter and decrease during the summer. However, the question remains, does one cause the other to occur?

IM2.3.5 Understand the effects of outliers on correlation coefficients, on the least squares regression line, and on the interpretations of correlation coefficients and regression lines in real-life contexts.

Example: In the example in indicator 3.1, consider the effect of adding a student who scores a 3 on the entrance test but received a 4 for the GPA at the end of the freshman year. This outlier would significantly lower the correlation coefficient to 0.386 and change the regression line to $y = 0.75 + 0.371x$. No longer does this data show a significant linear relationship, which, if used, would lead to wrong conclusions for the director of admissions.

Standard 4

Probability

Students construct probability distributions, understand fundamental probability concepts, and use counting principles.

IM2.4.1 Construct a probability distribution by simulation and use it to understand and analyze the probabilistic situation.

Example: Jamie is a basketball player who makes 70 percent of her free throws over the course of a season. In a key game, Jamie shoots 10 free throws. What is the probability she makes five of her free throws? Simulate 10 free throws shot independently using random numbers from 1 to 10 with 1 to 7 representing a success and record 100 trials.

IM2.4.2 Explore the geometric, or waiting-time, distribution.

Example: A card is drawn from a deck of cards. Observe the card and replace the card within the deck. What is the probability you will observe a jack on the sixth attempt?

$P(X = 6) = (12/13)^5 (1/13) = .0516$

IM2.4.3 Understand fundamental concepts of probability (e.g., independent events, multiplication rule, and expected value).

Example: Decide if the following events are independent: Bob draws a card from a standard deck of cards, replaces it, and then draws a second card. Explain your answer.

IM2.4.4 Understand and apply counting principles to compute combinations and permutations.

Example: There are 5 students who work in a bookshop. If the bookshop needs 3 people to operate, how many days straight could you work without the same group of students working twice?

IM2.4.5 Use the basic counting principle, combinations, and permutations to compute probabilities.

Example: You are on a chess team made up of 15 players. What is the probability that you will be chosen if a 3-person team is selected at random?

Standard 5

Discrete Mathematics

Students use graphs and networks as mathematical models and use matrices to solve problems.

IM2.5.1 Experience in mathematical modeling by building and using vertex-edge graph models to solve problems in a variety of real-world settings.

Example: Five students are to play in a ping-pong tournament. Each student is to play all the other players once. Draw a vertex-edge graph to show the winners of each match by drawing an arrow from the winner to the defeated student.

IM2.5.2 Develop the skill of algorithmic problem solving: designing, using, and analyzing systematic procedures for problem solving.

Example: Sarah wants to fly her aircraft from Maui to a remote island off its shore. To determine the amount of gas needed for a round trip, she must know how far the island is from Maui. Given the coordinates of the island and Maui, how many kilometers will she travel round trip?

IM2.5.3 **Optimize networks in different ways and in different contexts by finding minimal spanning trees, shortest paths, and Hamiltonian paths.**

Example: A family is planning a trip to see the Grand Canyon, Wisconsin Dells, Yellowstone National Park, Pikes Peak, Little Big Horn, and Mount Rushmore. The family wants to determine a path with the least amount of time on the road. What would be the best path to take?

IM2.5.4 **Use matrices to organize and display data in a variety of real-world settings.**

Example: Develop a matrix for the ping-pong tournament in the example in indicator 5.1 to rank each of the five students with the results from their win/loss records.

IM2.5.5 **Develop mathematical modeling skills by building matrix models and then apply the models to solve problems.**

Example: To prepare for a dance, a school needs to rent 100 chairs, four large tables, and 10 punch bowls. Rental prices were collected from two rental shops with the following matrix representing the two rental shops:

	R_1	R_2
Chairs	\$2	\$2.50
Tables	\$20	\$15
Bowls	\$6	\$4

Which rental shop, R_1 or R_2, has the lowest price for the group of items?

IM2.5.6 **Apply matrix operations to solve problems (i.e., row sums, scalar multiplication, addition, subtraction, and matrix multiplication).**

Example: Use matrix multiplication to solve the problem in indicator 5.5.

Solve $[100 \quad 4 \quad 10] \bullet \begin{bmatrix} 2 & 2.5 \\ 20 & 15 \\ 6 & 4 \end{bmatrix} = [340 \quad 350]$.

IM2.5.7 **Use matrices and inverse matrices to answer questions that involve systems of linear equations.**

Example: Solve the system of equations using matrices:
$2x + 20y + 6z = 340$
$2.5x + 15y + 4z = 350$

IM2.5.8 **Build and use matrix representations to model polygons, transformations, and computer animations.**

Example: Transform the following matrix, which represents the points on a triangle, to reflect across the y-axis to resemble a spinning effect.

Triangle $ABC = \begin{bmatrix} -2 & 2 & 0 \\ -1 & 1 & 5 \end{bmatrix}$

Standard 6

Trigonometry

Students apply trigonometric ratios to right triangles.

IM2.6.1 Explore properties and applications of the sine, cosine, and tangent ratios for the lengths of sides of right triangles.

Example: A farmer needs to change a bulb that is 35 feet high on the side of his barn. Because of an automatic water dispenser at the base of the barn, directly under the light, the angle at which the ladder will be placed on the ground to the barn is a maximum of 70°. The farmer only has a 30-foot ladder. Assuming the farmer is six feet tall, is the ladder tall enough for the farmer to change the light bulb?

Standard 7

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems and develop and evaluate mathematical arguments and proofs.

IM2.7.1 Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Solve $3x + 5 = 2x - 1$, explaining why you can take each step.

IM2.7.2 Make conjectures about geometric ideas. Distinguish between information that supports a conjecture and the proof of a conjecture.

Example: Calculate the ratios of side lengths in several different-sized triangles with angles of 90°, 50°, and 40°. What do you notice about the ratios? How might you prove that your observation is true (or show that it is false)?

IM2.7.3 Write and interpret statements of the form "if – then" and "if and only if."

Example: Decide whether this statement is true: "If today is Sunday, then we have school tomorrow."

IM2.7.4 State, use, and examine the validity of the converse, inverse, and contrapositive of "if – then" statements.

Example: In the example in indicator 7.3, write the converse of the statement.

IM2.7.5 Write geometric proofs, including proofs by contradiction and proofs involving coordinate geometry. Use and compare a variety of ways to present deductive proofs, such as flow charts, paragraphs, and two-column and indirect proofs.

Example: In triangle LMN, $LM = LN$. Prove that $\angle LMN \cong \angle LNM$.

IM2.7.6 Perform basic constructions, describing and justifying the procedures used. Distinguish between constructing and drawing geometric figures.

Example: Construct a line parallel to a given line through a given point not on the line, explaining and justifying each step.

IM2

IM2.7.7 **Decide if a given algebraic statement is true always, sometimes, or never (statements involving quadratic expressions).**

Example: Is the statement $x^2 - 4 \geq 0$ true for all x, for some x, or for no x? Explain.

IM2.7.8 **Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution as the original. Understand that similar logic applies to solving systems of equations simultaneously.**

Example: Try "solving" the equation $4x - 9 = 5 - 2(8 - 2x)$ and explain what went wrong.

IM2.7.9 **Use counterexamples to show that statements are false.**

Example: Show by an example that this statement is false: two triangles with the same area and same perimeter are congruent.

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Integrated Mathematics III:

Standard 1 — Algebra and Functions

Students solve linear inequalities by using order properties of the real numbers and they operate with polynomials — adding, subtracting, multiplying, dividing, and raising to powers — and find factors of polynomials, learning special techniques for factoring quadratics. Students simplify algebraic fractions, using what they have learned about factoring polynomials. They solve algebraic proportions. They solve quadratic equations by using the formula, by factoring, and by completing the square. They also solve equations that contain radical expressions. Students recognize and graph polynomial, rational, and algebraic functions. They use a variety of methods to solve systems of up to three linear equations in up to three variables. Students write equations and draw graphs of conic sections (circle, ellipse, parabola, and hyperbola), thus relating an algebraic representation to a geometric one. They add, subtract, multiply, divide, and simplify algebraic fractions and solve equations involving algebraic fractions. Students understand the concepts of logarithmic and exponential functions. Students define the concepts of arithmetic and geometric sequences and series.

Standard 2 — Geometry and Measurement

Students relate geometry to algebra by using coordinate geometry to determine congruence, similarity, symmetry, and tessellations. They prove that triangles are congruent or similar. Students find the equation of a circle in the coordinate plane and describe and make regular and nonregular polyhedra (cube, pyramid, tetrahedron, octahedron, etc.). They understand the properties of congruent and similar solids.

Standard 3 — Data Analysis and Statistics

Students will apply basic ideas related to surveys, construct simulated sampling distributions, interpret margin of error and confidence intervals, and understand standard deviation.

Standard 4 — Probability

Students will apply the Addition Rule for mutually exclusive events.

Standard 5 — Discrete Mathematics

Students will use iteration and recursion to solve problems.

Standard 6 — Trigonometry

Students will use the Law of Sines and Law of Cosines to find measures of sides and angles in triangles. They will also analyze families of trigonometric functions.

Standard 7 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. Students also apply these skills to constructing logical arguments and learn about inductive and deductive reasoning, as well as how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Algebra and Functions

Students solve inequalities, quadratic equations, and systems of equations. They graph polynomial, rational, algebraic, and piece-wise defined functions. They graph and write the equations of conic sections and compute with and factor polynomials and algebraic fractions. They solve problems involving exponential and logarithmic expressions, as well as define and use arithmetic and geometric sequences and series.

IM3.1.1 Solve combined linear inequalities.

Example: Solve the inequalities $-7 < 3x + 5 < 11$.

IM3.1.2 Use a graph to find the solution set of a pair of linear inequalities in two variables.

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

IM3.1.3 Find a common monomial factor in a polynomial.

Example: Factor $36xy^2 + 18xy^4 - 12x^2y^4$.

IM3.1.4 Factor the difference of two squares and other quadratics.

Example: Factor $4x^2 - 25$ and $2x^2 - 7x + 3$.

IM3.1.5 Simplify algebraic ratios.

Example: Simplify $\frac{x^2 - 16}{x^2 + 4x}$.

IM3.1.6 Solve algebraic proportions.

Example: Create a tutorial to be posted to the school's Web site to instruct beginning students in the steps involved in solving an algebraic proportion. Use $\frac{x + 5}{4} = \frac{3x + 5}{7}$ as an example.

IM3.1.7 Solve quadratic equations by factoring.

Example: Solve the equation $x^2 - 3x + 2 = 0$ by factoring.

IM3.1.8 Solve quadratic equations in which a perfect square equals a constant.

Example: Solve the equation $(x - 7)^2 = 64$.

IM3.1.9 Complete the square to solve quadratic equations.

Example: Solve the equation $x^2 - 7x + 9 = 0$ by completing the square.

IM3.1.10 Derive the quadratic formula by completing the square.

Example: Prove that the equation $ax^2 + bx + c = 0$ has solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

IM3.1.11 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x + 6} = x$.

IM3.1.12 Recognize and graph various types of functions, including polynomial, rational, and algebraic functions.

Example: Draw the graphs of the functions $y = x^4 - x^2$, $y = \frac{7}{x - 2}$, and $y = \sqrt{x + 2}$.

IM3.1.13 Use function notation. Add, subtract, multiply, and divide pairs of functions.

Example: Let $f(x) = 7x + 2$ and $g(x) = x^2$. Find the value of $f(x) \bullet g(x)$.

IM3.1.14 Understand composition of functions and combine functions by composition.

Example: Let $f(x) = x^3$ and $g(x) = x - 2$. Find $f(g(x))$.

IM3

IM3.1.15 Graph relations and functions with and without graphing technology.

Example: Draw the graph of $y = x^3 - 3x^2 - x + 3$.

IM3.1.16 Find the zeros of a function.

Example: In the example in indicator 1.15, find the zeros of the function; i.e., find x when $y = 0$.

IM3.1.17 Solve an inequality by examining the graph.

Example: Find the solution for $x^3 - 3x^2 - x + 3 < 0$ by graphing $y = x^3 - 3x^2 - x + 3$.

IM3.1.18 Graph functions defined piece-wise.

Example: Sketch the graph of $f(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x > 0 \end{cases}$.

IM3.1.19 Graph absolute value equations and inequalities.

Example: Draw the graph of $y = 2x - 5$ and use that graph to draw the graph of $y = |2x - 5|$.

IM3.1.20 Use substitution, elimination, and matrices to solve systems of two or three equations in two or three variables.

Example: Solve the system of equations: $x - 2y + 3z = 5, x + 3z = 11, 5y - 6z = 9$.

IM3.1.21 Use systems of linear equations and inequalities to solve word problems.

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

IM3.1.22 Define complex numbers and perform basic operations with them.

Example: Multiply $7 - 4i$ and $10 + 6i$.

IM3.1.23 Understand how real and complex numbers are related, including plotting complex numbers as points in the plane.

Example: Plot the points corresponding to $3 - 2i$ and $1 + 4i$. Add these complex numbers and plot the result. How is this point related to the other two?

IM3.1.24 Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

IM3.1.25 Solve word problems using quadratic equations.

Example: You have 100 feet of fencing to make three sides of a rectangular area using an existing straight fence as the fourth side. Construct a formula in a spreadsheet to determine the area you can enclose and use the spreadsheet to make a conjecture about the maximum area possible. Prove (or disprove) your conjecture by solving an appropriate quadratic equation.

IM3.1.26 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x + 9} = 9 - \sqrt{x}$.

IM3.1.27 Solve pairs of equations, one quadratic and one linear or both quadratic.

Example: Solve the system of equations $y = x^2 - 5x + 1, x + y + 2 = 0$.

IM3.1.28 Write the equations of conic sections (circle, ellipse, parabola, and hyperbola).

Example: Write an equation for a parabola with focus $(2, 3)$ and directrix $y = 1$.

IM3.1.29 Graph conic sections.

Example: Graph the circle described by the equation $(x + 4)^2 + (y - 1)^2 = 9$.

IM3.1.30 Understand the binomial theorem and use it to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$.

IM3.1.31 Divide polynomials by others of lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

IM3.1.32 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

IM3.1.33 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

IM3.1.34 Use polynomial equations to solve word problems.

Example: You want to make an open-top box with a volume of 500 square inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Find the possible dimensions of the box.

IM3.1.35 Write a polynomial equation given its solutions.

Example: Write an equation that has solutions $x = 2, x = 5i$ and $x = -5i$.

IM3.1.36 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

IM3.1.37 Understand and use negative and fractional exponents.

Example: Simplify $(2a^{-2}b^3)^4 (4a^3b^{-1})^{-2}$.

IM3.1.38 Add, subtract, multiply, divide, and simplify algebraic fractions.

Example: Simplify $\frac{x^2 - 4}{x^5} \div \frac{x^3 - 8}{x^8}$.

IM3.1.39 Simplify complex fractions.

Example: Simplify $\left(\frac{5}{x - 2} + \frac{2}{x + 3}\right) \div \left(\frac{1}{x + 3} + \frac{7}{x - 2}\right)$.

IM3.1.40 Solve equations involving algebraic fractions.

Example: Solve $\frac{10}{n} + \frac{5}{n^2 - 4} = \frac{7}{n - 2}$.

IM3.1.41 Solve word problems involving fractional equations.

Example: Two students, working independently, can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together?

IM3.1.42 Solve problems of direct, inverse, and joint variation.

Example: One day your drive to work takes 10 minutes and you average 30 mph. The next day the drive takes 15 minutes. What is your average speed that day?

IM3.1.43 Prove simple laws of logarithms.

Example: Use the fact that $a^x \bullet a^y = a^{x+y}$ to show that $\log_a(pq) = \log_a p + \log_a q$.

IM3.1.44 Understand and use the inverse relationship between exponents and logarithms.

Example: Find the value of $\log_{10}(10^7)$.

IM3.1.45 Solve logarithmic and exponential equations and inequalities.

Example: Solve the equation $\log_2 x = 5$.

IM3.1.46 Use the definition of logarithms to convert logarithms from one base to another.

Example: Write $\log_{10} 75$ as a logarithm to base 2.

IM3.1.47 Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values.

Example: Simplify $\log_3 81$.

IM3.1.48 Use calculators to find decimal approximations of natural and common logarithmic numeric expressions.

Example: Find a decimal approximation for ln 500.

IM3.1.49 Solve word problems involving applications of exponential functions to growth and decay.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

IM3.1.50 Define arithmetic and geometric sequences and series.

Example: What type of sequence is 10, 100, 1,000, 10,000, …?

IM3.1.51 Find specified terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15, … .

IM3.1.52 Find partial sums of arithmetic and geometric series.

Example: In the example in 1.51, find the sum of the first 10 terms.

IM3.1.53 Solve word problems involving applications of sequences and series.

Example: You have on a Petri dish 1 square millimeter of a mold that doubles in size each day. What area will it cover after a month?

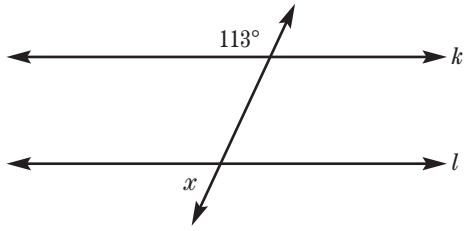
Standard 2

Geometry and Measurement

Students describe and use parallel and perpendicular lines. They use coordinate geometry and prove that triangles are congruent or similar. They find the equation of a circle in the coordinate plane and describe and use properties of solids.

IM3.2.1 Understand and use the relationships between special pairs of angles formed by parallel lines and transversals.

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.



IM3.2.2 Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Example: Find an equation of a line perpendicular to $y = 4x - 2$.

IM3.2.3 Use properties of congruent and similar polygons to solve problems.

Example: Divide a regular hexagon into triangles by joining the center to each vertex. Show that these triangles are all the same size and shape and find the sizes of the interior angles of the hexagon.

IM3.2.4 Use coordinate geometry to prove properties of polygons such as regularity, congruence, and similarity.

Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6), and (1, 5) a square?

IM3.2.5 Describe, classify, and understand relationships among the quadrilaterals square, rectangles, rhombus, parallelogram, trapezoid, and kite.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid, and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

IM3.2.6 Use coordinate geometry to prove properties of quadrilaterals such as regularity, congruence, and similarity.

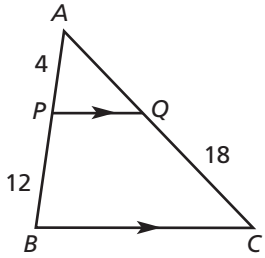
Example: Is rectangle *ABCD* with vertices at (0, 0), (4, 0), (4, 2), (0, 2) congruent to rectangle *PQRS* with vertices at (-2, -1), (2, -1), (2, 1), (-2, 1)?

IM3.2.7 Construct triangles congruent to given triangles.

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

IM3.2.8 Prove and apply theorems involving segments divided proportionally.

Example: In triangle ABC, $\overline{PQ}$ is parallel to $\overline{BC}$. What is the length of $\overline{AQ}$?



IM3.2.9 Prove that triangles are congruent or similar and use the concept of corresponding parts of congruent triangles.

Example: In the example in indicator 2.8, prove that triangles ABC and APQ are similar.

IM3.2.10 Use coordinate geometry to prove properties of triangles such as regularity, congruence, and similarity.

Example: Draw a triangle with vertices at (1, 3), (2, 5), and (6, 1). Draw another triangle with vertices at (-3, -1), (-2, 1), and (2, -3). Are these triangles the same shape and size?

IM3.2.11 Find the equation of a circle in the coordinate plane in terms of its center and radius.

Example: Find the equation of the circle with radius 10 and center (6, -3).

IM3.2.12 Describe and make regular and nonregular polyhedra.

Example: Is a cube a regular polyhedron? Explain why or why not.

IM3.2.13 Describe the polyhedron that can be made from a given net (or pattern). Describe the net for a given polygon.

Example: Make a net for a tetrahedron out of poster board and fold it up to make the tetrahedron.

IM3.2.14 Identify and know properties of congruent and similar solids.

Example: Explain how the surface area and volume of similar cylinders are related.

IM3.2.15 Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.

Example: An ice cube is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the ice cube?

Standard 3

Data Analysis and Statistics

Students design and interpret surveys, use sampling distributions, and understand standard deviation.

IM3.3.1 Understand and apply basic ideas related to the design and interpretation of surveys, such as background information, random sampling, and bias.

Example: Explain how to design a survey that is random and not biased in nature.

IM3.3.2 Construct simulated sampling distributions of sample proportions and use sampling distributions to identify which proportions are likely to be found in a sample of a given size.

Example: About 30 percent of the students at a school are on the honor roll. If you took a random sample of 30 students, what range of students would be likely to be on the honor roll?

IM3.3.3 Construct and interpret margin of error and confidence intervals for population proportions.

Example: In a random sample of 40 people at a restaurant, 13 of them say that they prefer black coffee. Use a 90 percent confidence interval to find approximately what percent of all patrons at the restaurant prefer black coffee.

IM3.3.4 Understand the standard deviation as a measure of variability in a distribution.

Example: Explain how the values vary about the mean. Given a set of test scores: 99, 96, 94, 93, 90, 88, 86, 77, 70, 68, find the mean and standard deviation.

Standard 4

Probability

Students apply the Addition Rule for mutually exclusive events.

IM3.4.1 Understand and apply the Addition Rule for mutually exclusive events.

Example: Nicki is an 80 percent free-throw shooter. She is in a two-shot foul situation. Find the probability she will make only one free throw.

Standard 5

Discrete Mathematics

Students use iteration and recursion to solve problems.

IM3.5.1 Use iteration and recursion as tools to represent, analyze, and solve problems involving sequential change.

Example: There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of the year. Find the population in five years. Also, find the long-term population.

IM3.5.2 Explore function iteration and, in the process, informally introduce function composition.

Example: Iterate the function $f(x) = 3x + 2$ starting with $x = 1$. Find the first four iterates.

IM3.5.3 Understand and apply recursion equations, particularly combined recursion equations of the form $A_n = rA_{n\text{-}1} + b$.

Example: Write a recursion equation to model the following situation: You buy a $10,000 car with an annual interest rate of 6 percent and make a monthly payment of $250. Find the number of months needed to pay off the car.

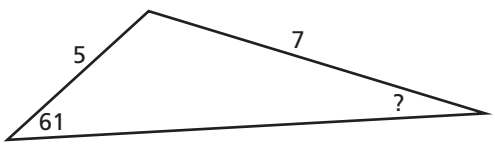
Standard 6

Trigonometry

Students use the Law of Sines and the Law of Cosines to solve problems. They analyze families of trigonometric functions.

IM3.6.1 Find the measures of sides and angles in triangles using the Law of Sines.

Example: Find the measure of the missing angle shown.



IM3.6.2 Find the measures of sides and angles in triangles using the Law of Cosines.

Example: In triangle ABC, find the length of side c if side $a = 10$, $b = 26$, and $\angle C = 96°$.

IM3.6.3 Compare and contrast families of trigonometric functions.

Example: Draw a graph of $f(x) = \sin x$, and $f(x) = \cos x$, what are the similarities and differences of the two graphs?

Standard 7

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems and develop and evaluate mathematical arguments and proofs.

IM3.7.1 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: A student solving the equation $\sqrt{(x+6)} = x$ comes up with the solution set $\{-2, 3\}$. Explain why $\{-2, 3\}$ is not the solution set to this equation, and why the "check" step is essential in solving the equation.

IM3.7.2 Decide whether a given algebraic statement is true always, sometimes, or never (statements involving rational or radical expressions or logarithmic or exponential functions).

Example: Is the statement $(a^x)^y = a^{xy}$ true for all x, for some x, or for no x? Explain your answer.

IM3.7.3 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

Example: What type of reasoning are you using when you look for a pattern?

IM3.7.4 Identify the hypothesis and conclusion in a logical deduction.

Example: What is the hypothesis and conclusion in this argument: If there is a number x such that $2x + 1 = 7$, then $x = 3$?

IM3.7.5 Use counterexamples to show that statements are false, recognizing that a single counterexample is sufficient to prove a general statement false.

Example: Show by an example that this statement is false: The product of two complex numbers is never a real number.

IM3.7.6 Use the properties of number systems and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Simplify $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$, explaining why you can take each step.

IM3.7.7 Identify and give examples of undefined terms, axioms, and theorems, and inductive and deductive proofs.

Example: Do you prove axioms from theorems or theorems from axioms?

IM3.7.8 Construct logical arguments, judge their validity, and give counterexamples to disprove statements.

Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent.

NOTES

Pre-Calculus/Trigonometry

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Pre-Calculus/Trigonometry:

Standard 1 — Relations and Functions

Students recognize and graph polynomial, rational, algebraic, and absolute value functions and use them to solve word problems. They understand the concepts of domain, range, intercept, zero, pole, asymptote, and point of discontinuity. They define and find inverse functions, describe symmetries of graphs, and apply transformations to functions. They understand the concept of defining a function parametrically and apply it to drawing graphs. They write equations of conic sections in standard form to find their geometric properties.

Standard 2 — Logarithmic and Exponential Functions

Students solve word problems involving logarithmic and exponential functions. They draw and analyze graphs of logarithmic and exponential functions, including finding domain, range, intercepts, and asymptotes. They define and find inverse functions for both logarithmic and exponential functions.

Standard 3 — Trigonometry in Triangles

Students understand how trigonometric functions relate to right triangles and solve word problems involving right and oblique triangles. They understand and apply the laws of sines and cosines. They use trigonometry to find the area of a triangle from two sides and the included angle.

Standard 4 — Trigonometric Functions

Students extend the definitions of the trigonometric functions beyond right triangles using the unit circle and they measure angles in radians as well as degrees. They draw and analyze graphs of trigonometric functions (including finding period, amplitude, and phase shift) and use them to solve word problems. They define and graph inverse trigonometric functions and find values of both trigonometric and inverse trigonometric functions. They also relate the slope of a line to the tangent of the angle the line makes with the x-axis.

Standard 5 — Trigonometric Identities and Equations

Students know basic trigonometric identities derived from the definitions and use them to prove other results. In particular, they understand and use the addition, double-angle, and half-angle formulas. They solve trigonometric equations and apply the equations to word problems.

Standard 6 — Polar Coordinates and Complex Numbers

Students define and use polar coordinates, understanding their relationship with Cartesian coordinates. They translate equations in Cartesian coordinates into polar coordinates and graph equations in the polar coordinate plane. They understand complex numbers and convert them to trigonometric form. They multiply complex numbers in trigonometric form and prove and use De Moivre's Theorem.

Standard 7 — Sequences and Series

Students prove the formulas for the sums of arithmetic series and for finite and infinite geometric series, using summation notation and applying the results to word problems. They understand the concept of recursion and define sequences using it. They develop the concept of the limit of a sequence or a function and apply it to problems of convergence and divergence.

Standard 8 — Data Analysis

Students understand the median fit and least squares regression methods and apply them to linear modeling. They calculate and interpret correlation coefficients, using them to evaluate lines of best fit. They model data with various nonlinear functions, such as quadratic, exponential, and power functions.

Standard 9 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn how to use the mathematical induction to prove results.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Pre-Calculus/Trigonometry

Pre-Calculus and Trigonometry are sometimes taught separately as one-semester courses, but are more often taught as a single course. If taught separately, the courses should consist of the following standards:

Pre-Calculus

- Standard 1 Relations and Functions
- Standard 2 Logarithmic and Exponential Functions
- Standard 7 Sequences and Series
- Standard 8 Data Analysis
- Standard 9 Mathematical Reasoning and Problem Solving

Trigonometry

- Standard 3 Trigonometry in Triangles
- Standard 4 Trigonometric Functions
- Standard 5 Trigonometric Identities and Equations
- Standard 6 Polar Coordinates and Complex Numbers
- Standard 9 Mathematical Reasoning and Problem Solving

PC

Standard 1

Relations and Functions

Students use polynomial, rational, and algebraic functions to write functions and draw graphs to solve word problems, to find composite and inverse functions, and to analyze functions and graphs. They analyze and graph circles, ellipses, parabolas, and hyperbolas.

PC.1.1 Recognize and graph various types of functions, including polynomial, rational, algebraic, and absolute value functions. Use paper and pencil methods and graphing calculators.

Example: Draw the graphs of the functions $y = x^5 - 2x^3 - 5x^2$, $y = \frac{2x-1}{3x+2}$, and $y = \sqrt{(x+2)(x-5)}$.

PC.1.2 Find domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions. Use paper and pencil methods and graphing calculators.

Example: Let $R(x) = \frac{1}{\sqrt{x-2}}$. Find the domain of $R(x)$ — i.e., the values of x for which $R(x)$ is defined. Also find the range, zeros, and asymptotes of $R(x)$.

PC.1.3 Model and solve word problems using functions and equations.

Example: You are on the committee for planning the prom and need to decide what to charge for tickets. Last year you charged $5.00 and 400 people bought tickets. Earlier experiences suggest that for every 10¢ decrease in price you will sell 50 extra tickets. Use a spreadsheet and write a function to show how the amount of money in ticket sales depends on the number of 10¢ decreases in price. Construct a graph that shows the price and gross receipts. What is the optimum price you should set for the tickets?

PC.1.4 Define, find, and check inverse functions.

Example: Find the inverse function of $h(x) = (x-2)^3$.

PC.1.5 Describe the symmetry of the graph of a function.

Example: Describe the symmetries of the functions x, x^2, x^3, and x^4.

PC.1.6 Decide if functions are even or odd.

Example: Is the function $\tan x$ even, odd, or neither? Explain your answer.

PC.1.7 Apply transformations to functions.

Example: Explain how you can obtain the graph of $g(x) = -|2(x+3)^2 - 2|$ from the graph of $f(x) = x^2$.

PC.1.8 Understand curves defined parametrically and draw their graphs.

Example: Draw the graph of the function $y = f(x)$, where $x = 3t + 1$ and $y = 2t^2 - 5$ for a parameter t.

PC.1.9 Compare relative magnitudes of functions and their rates of change.

Example: Contrast the growth of $y = x^2$ and $y = 2^x$.

PC.1.10 Write the equations of conic sections in standard form (completing the square and using translations as necessary), in order to find the type of conic section and to find its geometric properties (foci, asymptotes, eccentricity, etc.).

Example: Write the equation $x^2 + y^2 - 10x - 6y - 25 = 0$ in standard form. Decide what kind of conic it is and find its foci, asymptotes, and eccentricity as appropriate.

PC

Standard 2

Logarithmic and Exponential Functions

Students solve word problems involving logarithmic and exponential functions. They draw and analyze graphs and find inverse functions.

PC.2.1 Solve word problems involving applications of logarithmic and exponential functions.

Example: The amount A gm of a radioactive element after t years is given by the formula $A(t) = 100\,e^{-0.02t}$. Find t when the amount is 50 gm, 25 gm, and 12.5 gm. What do you notice about these time periods?

PC.2.2 Find the domain, range, intercepts, and asymptotes of logarithmic and exponential functions.

Example: For the function $L(x) = \log_{10}(x - 4)$, find its domain, range, x-intercept, and asymptote.

PC.2.3 Draw and analyze graphs of logarithmic and exponential functions.

Example: In the last example, draw the graph of $L(x)$.

PC.2.4 Define, find, and check inverse functions of logarithmic and exponential functions.

Example: Find the inverse of $f(x) = 3e^{2x}$.

Standard 3

Trigonometry in Triangles

Students define trigonometric functions using right triangles. They solve word problems and apply the laws of sines and cosines.

PC.3.1 Solve word problems involving right and oblique triangles.

Example: You want to find the width of a river that you cannot cross. You decide to use a tall tree on the other bank as a landmark. From a position directly opposite the tree, you measure 50 m along the bank. From that point, the tree is in a direction at 37° to your 50 m line. How wide is the river?

PC.3.2 Apply the laws of sines and cosines to solving problems.

Example: You want to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78°. From the second position, the angle between the mountain and the first position is 53°. How far is the mountain from each position?

PC.3.3 Find the area of a triangle given two sides and the angle between them.

Example: Calculate the area of a triangle with sides of length 8 cm and 6 cm enclosing an angle of 60°.

Standard 4

Trigonometric Functions

Students define trigonometric functions using the unit circle and use degrees and radians. They draw and analyze graphs, find inverse functions, and solve word problems.

PC.4.1 Define sine and cosine using the unit circle.

Example: Find the acute angle A for which $\sin 150^\circ = \sin A$.

PC.4.2 Convert between degree and radian measures.

Example: Convert 90º, 45º, and 30º to radians.

PC.4.3 Learn exact sine, cosine, and tangent values for 0, $\pi/2$, $\pi/3$, $\pi/4$, $\pi/6$, and multiples of π. Use those values to find other trigonometric values.

Example: Find the values of $\cos \pi/2$, $\tan 3\pi/4$, $\csc 2\pi/3$, $\sin^{-1} \sqrt{3}/2$, and $\sin 3\pi$.

PC.4.4 Solve word problems involving applications of trigonometric functions.

Example: In Indiana, the day length in hours varies through the year in a sine wave. The longest day of 14 hours is on Day 175 and the shortest day of 10 hours is on Day 355. Sketch a graph of this function and find its formula. Which other day has the same length as July 4?

PC.4.5 Define and graph trigonometric functions (i.e., sine, cosine, tangent, cotangent, secant, cosecant).

Example: Graph $y = \sin x$ and $y = \cos x$, and compare their graphs.

PC.4.6 Find domain, range, intercepts, periods, amplitudes, and asymptotes of trigonometric functions.

Example: Find the asymptotes of $\tan x$ and find its domain.

PC.4.7 Draw and analyze graphs of translations of trigonometric functions, including period, amplitude, and phase shift.

Example: Draw the graph of $y = 5 + \sin (x - \pi/3)$.

PC.4.8 Define and graph inverse trigonometric functions.

Example: Graph $f(x) = \sin^{-1} x$.

PC.4.9 Find values of trigonometric and inverse trigonometric functions.

Example: Find the values of $\sin \pi/2$ and $\tan^{-1} \sqrt{3}$.

PC.4.10 Know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.

Example: Use a right triangle to show that the slope of a line at 135º to the x-axis is -1.

PC.4.11 Make connections between right triangle ratios, trigonometric functions, and circular functions.

Example: Angle A is a 60º angle of a right triangle with a hypotenuse of length 14 and a shortest side of length 7. Find the exact sine, cosine, and tangent of angle A. Find the real numbers x, $0 < x < 2\pi$, with exactly the same sine, cosine, and tangent values.

PC

Standard 5

Trigonometric Identities and Equations

Students prove trigonometric identities, solve trigonometric equations, and solve word problems.

PC.5.1 Know the basic trigonometric identity $\cos^2 x + \sin^2 x = 1$ and prove that it is equivalent to the Pythagorean Theorem.

Example: Use a right triangle to show that $\cos^2 x + \sin^2 x = 1$.

PC.5.2 Use basic trigonometric identities to verify other identities and simplify expressions.

Example: Show that $\frac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x$.

PC.5.3 Understand and use the addition formulas for sines, cosines, and tangents.

Example: Prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and use it to find a formula for $\sin 2x$.

PC.5.4 Understand and use the half-angle and double-angle formulas for sines, cosines, and tangents.

Example: Prove that $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$.

PC.5.5 Solve trigonometric equations.

Example: Solve $3 \sin 2x = 1$ for x between 0 and 2π.

PC.5.6 Solve word problems involving applications of trigonometric equations.

Example: In the example about day length in Standard 4, for how long in winter is there less than 11 hours of daylight?

Standard 6

Polar Coordinates and Complex Numbers

Students define polar coordinates and complex numbers and understand their connection with trigonometric functions.

PC.6.1 Define polar coordinates and relate polar coordinates to Cartesian coordinates.

Example: Convert the polar coordinates $(2, \frac{\pi}{3})$ to (x, y) form.

PC.6.2 Represent equations given in rectangular coordinates in terms of polar coordinates.

Example: Represent the equation $x^2 + y^2 = 4$ in terms of polar coordinates.

PC.6.3 Graph equations in the polar coordinate plane.

Example: Graph $y = 1 - \cos\theta$.

PC.6.4 Define complex numbers, convert complex numbers to trigonometric form, and multiply complex numbers in trigonometric form.

Example: Write $3 + 3i$ and $2 - 4i$ in trigonometric form and then multiply the results.

PC.6.5 State, prove, and use De Moivre's Theorem.

Example: Simplify $(1 - i)^{23}$.

Standard 7

Sequences and Series

Students define and use arithmetic and geometric sequences and series, understand the concept of a limit, and solve word problems.

PC.7.1 Understand and use summation notation.

Example: Write the terms of $\sum_{1}^{5} n^2$.

PC.7.2 Find sums of infinite geometric series.

Example: Find the sum of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$.

PC.7.3 Prove and use the sum formulas for arithmetic series and for finite and infinite geometric series.

Example: Prove that $a + ar + ar^2 + ar^3 + ar^4 + \ldots = a / (1 - r)$.

PC.7.4 Use recursion to describe a sequence.

Example: Write the first five terms of the Fibonacci sequence with $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$.

PC.7.5 Understand and use the concept of limit of a sequence or function as the independent variable approaches infinity or a number. Decide whether simple sequences converge or diverge.

Example: Find the limit as $n \to \infty$ of the sequence $\frac{2n - 1}{3n + 2}$ and the limit as $x \to 5$ of the function $\frac{x^2 - 5^2}{x - 5}$.

PC.7.6 Solve word problems involving applications of sequences and series.

Example: You put $100 in your bank account today, and then each day put half the amount of the previous day (always rounding to the nearest cent). Will you ever have $250 in your account?

Standard 8

Data Analysis

Students model data with linear and nonlinear functions.

PC.8.1 Find linear models using the median fit and least squares regression methods. Decide which model gives a better fit.

Example: Measure the wrist and neck size of each person in your class and make a scatterplot. Find the median fit line and the least squares regression line. Which line is a better fit? Explain your reasoning.

PC.8.2 Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a "best-fit" line.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

PC.8.3 Find a quadratic, exponential, logarithmic, power, or sinusoidal function to model a data set and explain the parameters of the model.

Example: Drop a ball and record the height of each bounce. Make a graph of the height (vertical axis) versus the bounce number (horizontal axis). Find an exponential function of the form $y = a \bullet b^x$ that fits the data and explain the implications of the parameters a and b in this experiment.

Standard 9

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

PC.9.1 Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, examining simpler problems, and working backwards.

Example: The half-life of carbon-14 is 5,730 years. The original concentration of carbon-14 in a living organism was 500 grams. How might you find the age of a fossil of that living organism with a carbon-14 concentration of 140 grams?

PC.9.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is about 10,000 years. Is his answer reasonable? Why or why not?

Students develop and evaluate mathematical arguments and proofs.

PC.9.3 Decide if a given algebraic statement is true always, sometimes, or never (statements involving rational or radical expressions, trigonometric, logarithmic or exponential functions).

Example: Is the statement $\sin 2x = 2 \sin x \cos x$ true for all x, for some x, or for no x? Explain your answer.

PC.9.4 Use the properties of number systems and order of operations to justify the steps of simplifying functions and solving equations.

Example: Simplify $\left(\frac{5}{x-2}+\frac{2}{x+3}\right) \div \left(\frac{1}{x+3}+\frac{7}{x-2}\right)$, explaining why you can take each step.

PC.9.5 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation, and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: A student solving the equation $x + \sqrt{x} - 30 = 0$ comes up with the solution set $\{25, 36\}$. Explain why $\{25, 36\}$ is not the solution set to this equation, and why the "check" step is essential in solving the equation.

PC.9.6 Define and use the mathematical induction method of proof.

Example: Prove De Moivre's Theorem using mathematical induction.

NOTES

PC

Probability and Statistics

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Probability and Statistics:

Standard 1 — Descriptive Statistics

Students create, compare, and evaluate data displays using such methods as histograms, cumulative distribution functions, and scatterplots. For these data, they calculate measures of central tendency (various kinds of mean, the median, and the mode) and their derivatives (range, quartiles, variance, and standard deviation).

Standard 2 — Probability

Students understand the counting principle, permutations, and combinations and use them to solve problems. They develop rules for finding probabilities of combined and complementary events. They understand and use conditional probability and the related Bayes' Theorem. They investigate probability distributions and calculate and interpret their means and variances. They use and apply the normal distribution, including using the central limit theorem.

Standard 3 — Statistical Inference

Students use confidence intervals and hypothesis tests of means and differences between means. They use the principle of least squares to find curves of best fit and they calculate and interpret correlation coefficients.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to investigating probability situations and applying them to distributions, confidence intervals, and hypothesis tests.

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical ideas includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Descriptive Statistics

Students gather and display data and use measures of central tendency and variability.

PS.1.1 Create, compare, and evaluate different graphic displays of the same data, using histograms, frequency polygons, cumulative frequency distribution functions, pie charts, scatterplots, stem-and-leaf plots, and box-and-whisker plots. Draw these by hand or use a computer spreadsheet program.

Example: Gather data to answer the question: Which area of the country has the highest high school dropout rate? Display your dropout data in various forms.

PS.1.2 Compute and use mean, median, mode, weighted mean, geometric mean, harmonic mean, range, quartiles, variance, and standard deviation.

Example: Use spreadsheet formulas to compute measures that summarize your dropout data by state.

Standard 2

Probability

Students solve problems involving the use of probability and probability distributions.

PS.2.1 Understand the counting principle, permutations, and combinations and use them to solve problems.

Example: A chess team has 5 players available. How many different teams of 3 could you have? If you are one of the players, what is the probability that you will be chosen when a team is selected at random? Show that the number of different teams of 2 players is the same as the number with 3 players. Explain why this is true.

PS.2.2 Understand and use the addition rule to calculate probabilities for mutually exclusive and nonmutually exclusive events.

Example: When you roll 3 dice, find the probability of obtaining an even total or a total less than 7.

PS.2.3 Understand and use the multiplication rule to calculate probabilities for independent and dependent events.

Example: When you roll 3 dice, find the probability of obtaining a 6, followed by an even number, followed by a 4.

PS.2.4 Calculate the probabilities of complementary events.

Example: Find the probability that at least two people in a group of 10 will have the same birthday.

PS.2.5 Understand conditional probability and Bayes' Theorem and use them to solve problems.

Example: The test for a disease is positive for 100% of those with the virus, but one person in 20,000 without the virus also tests positive. If one person in 1,000 actually has the virus, what is the probability that a person who tests positive does have the virus?

PS.2.6 Use discrete random variables and probability distributions, including the binomial and geometric distributions.

Example: When you flip a coin 5 times, the number of heads is 0, 1, 2, 3, 4, or 5. Find the probability of each number of heads and draw a histogram of the results.

PS.2.7 Compute and interpret the mean and variance of a probability distribution.

Example: In the last example, find the mean and variance of the number of heads.

PS.2.8 Use and apply the normal distribution.

Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that Joan's SAT score is greater than 550?

PS.2.9 Understand the central limit theorem and use it to solve problems.

Example: Compare the means of small samples of the lengths of words used in different sections of a newspaper (e.g., editorial, news, sports).

PS.2.10 Use other continuous random variables and probability distributions to solve problems.

Example: Compare graphs of the normal distribution with binomial, uniform, linear, and exponential distributions. Draw sets of graphs with a computer or graphing calculator.

Standard 3

Statistical Inference

Students use confidence intervals and hypothesis tests, fit curves to data, and calculate correlation coefficients.

PS.3.1 Compute and use confidence intervals to make estimates.

Example: Using the mean and standard deviation of a sample, calculate 95% confidence intervals for the true mean (assuming normality).

PS.3.2 Understand hypothesis tests of means and differences between means and use them to reach conclusions.

Example: Test a hypothetical mean using the mean and standard deviation of a sample (assuming normality).

PS.3.3 Use the principle of least squares to find the curve of best fit for a set of data.

Example: Measure the wrist and neck size of each person in your class and make a scatterplot. Find the median fit line and the least squares regression line. Which line is a better fit? Explain your reasoning.

PS.3.4 Calculate and interpret the correlation coefficient of a set of data.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

NOTES

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